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The fractal geometry of tidal-channel systems in the Dutch Wadden Sea

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Abstract

Horton's hierarchical and fractal analysis of channel circumference reveals that tidal-channel systems in the Dutch Wadden Sea have similar branching patterns. Channel systems have the same characteristics as three- to four-times branching networks. The branch lengths of these channels decrease logarithmically. The channel systems can be regarded as 'statistical self-similar fractal' networks, considering the natural variability in branch lengths and channel positions. Branching of channels does not continue below the 500 m scale. The channel-system circumference length is logarithmically related to the tidal prism and drainage area. The similarity of the channel systems, regardless of their size, relative amount of intertidal flats, and tidal amplitude, points to a self-organising nature. All processes depend on the feedback between morphology and hydrodynamics. At first sight, the channel systems can be regarded as an ebb-driven drainage network, governed by erosion. However, flood-dominated net sedimentation occurs in large parts of the drainage basins and modifies the ebb-driven network. The complex interaction of hydrodynamic and morphodynamic processes in tidal basins limits the applicability of process-based models. Behaviour-oriented modelling has a wide applicability and can be improved using the fractal geometry as the dynamical equilibrium morphology. The fractal-network geometry can also be used for stochastic reconstructions of fossil tidal-channel systems, when only limited observations are available.

Introduction

Tidal-channel systems occur world-wide in modern environments and in the fossil record (De Boer et al. 1988, Dalrymple 1992). They often consist of a large inlet channel, branching into smaller channels, which finally give way to (inter) tidal gullies (Van Straaten 1964, Oost & De Boer 1994). The complex branching patterns of tidal channel systems resemble fractal patterns, comparable to the fractal patterns of river networks (Mandelbrot 1982, Stark 1991). In general, fractals are characterised by 'power-law' relations between the scale of observation and the number, length or area of the phenomenon (Mandelbrot 1967 1982, Voss 1988, Turcotte 1992).

In this study, the geometry and possible fractal nature of tidal-channel systems in the Dutch Wadden Sea are investigated. Tides in the coupled tidal basins are semi-diurnal. In total, ten tidal-inlet systems accommodate the flooding and drainage of the area (Figure 1). The drainage area of the sub-basins ranges from 29 to 712 km^2 (Table 1; Louters & Gerritsen 1994). The tidal amplitude increases from 1.4 m in the western part to 3.5 m in the eastern part. The relative amount of inter-tidal flats in the back-barrier area varies from 17 to 82%. The sediments of the investigated areas are mainly very fine to fine sands. Coarse-grained sediments occur locally on channel floors. Mud is deposited on tidal flats near the mainland and in the vicinity of the watersheds. Dominantly muddy areas, as for example the Dollard, just east of the area of Figure 1, have not been incorporated in this research.

The methods used to study the tidal-channel systems are Horton's hierarchical analysis and fractal analysis. Analogues and differences of tidal-channel systems and river networks are discussed. Furthermore, the outcome of the analysis will be linked to the processes that govern the morphology of the tidal-channel system.

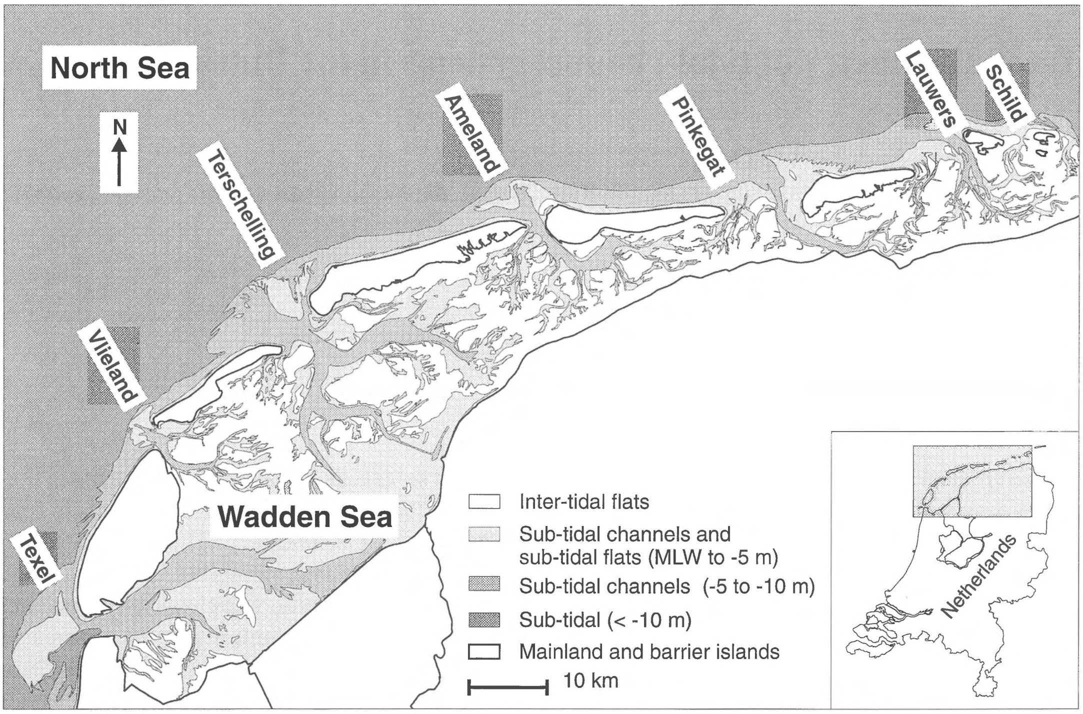

Figure 1. Tidal-channel systems in the Dutch Wadden Sea.

The results of the analyses enable predictions of the geometry of modern and fossil tidal-channel systems. The latter permits stochastic reconstructions of fossil tidal-channel systems. The results can also be applied in behaviour-oriented modelling of modern tidal systems.

Methods

The branching of channel systems is quantified at all scales by fractal analysis (Mandelbrot 1967). However, such analysis alone does not give a full characterisation of a network. Therefore, a morphological analysis was applied as well. This morphological analysis was first applied by Horton (1945), and defines the nature of the branching pattern, i.e., the number of branches of different size. The combination of these methods provides detailed insight into the fractal nature of the tidal-channel systems.

Horton's hierarchical analysis starts with the definition of the smallest channels as first-order channels. The confluence of two first-order channels defines a second-order channel, and so on (Figure 2). The confluence of a lower-order channel with a higher-order channel does not change the order of either of them. The analysis is accomplished by simply counting the number of first-, second- and higher-order channels. This has been done using the average low-water line of four inter-tidal systems (Ameland, Vlieland, Pinkegat and Lauwers, Figure 1), the –2.5 m line (with reference to the Dutch ordnance datum N.A.P.) of three channel systems (Terschelling, Texel and Lauwers), and the –5 m line of one sub-tidal system (Texel). A detailed map of the Dutch Wadden Sea, scale 1:100 000, compiled by Rijkswaterstaat (1980), was used.

In fractal analysis, the method used by Richardson (1961) and by Mandelbrot (1967, 1982) was applied. The length of the channel-system circumference is measured using a divider set at a fixed small length. This is repeated with the divider set at increasingly greater lengths (Figure 3). The amount of detail encountered decreases with the increase of divider length. The measured circumference length decreases therefore as well. The circumferences of the average low-water line of four (Ameland, Pinkegat, Terschelling and Vlieland), and the –2.5 m (N.A.P.) line of two (Terschelling and Texel) tidal-channel sys-

Table 1. Characteristics of tidal basins in the Dutch Wadden Sea (Louters & Gerritsen 1994). Channel-system circumference lengths from this study

Tidal-channel system	Average tidal prism (10^6 m^3)	Drainage area (km^2)	Inter-tidal flat area (km^2)	Channel-system circumference length (km)
Texel	1054	712	121	336
Vlieland	207	153	106	160
Terschelling	1078	668	323	360
Ameland	478	309	165	254
Pinkegat	100	65	42	69
Schild	31	29	26	26

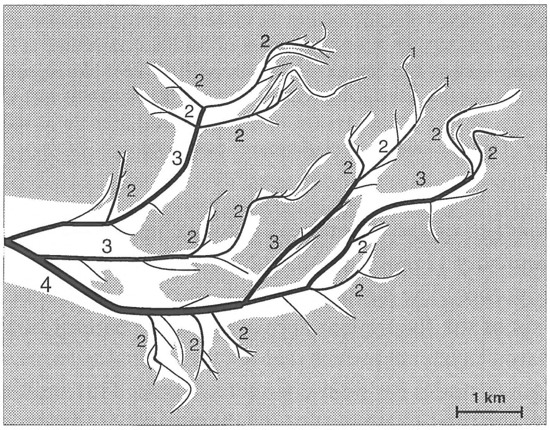

Figure 2. Horton's hierarchical analysis. The smallest channels are assigned to the first order. Two first-order channels merge into a second-order channel, and so on. A lower-order channel that joins a higher-order channel does not change the order of the latter.

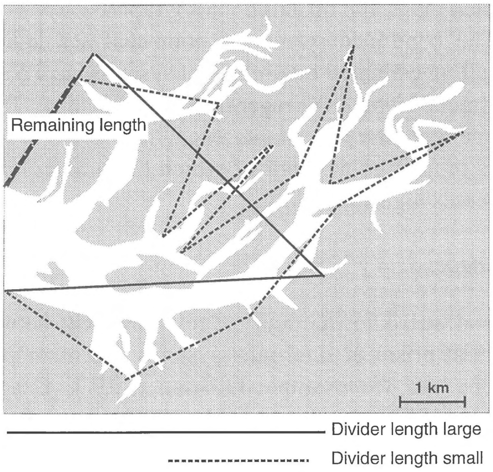

Figure 3. Fractal analysis. The channel-system circumference is measured with a divider set at increasing lengths. The greater the divider, the more details are missed, and the shorter the measured circumference.

tems were analysed. In this analysis the same map, scale 1:100 000 (Rijkswaterstaat 1980), was used.

In both analyses the quality of the map is essential, especially concerning the detail of small-scale features. The accuracy of a map generally decreases rapidly with decreasing scale. However, comparison with more detailed maps (sounding charts of 1:10 000) and aerial photographs reveals that the map (Rijkswaterstaat 1980) is accurate for the purpose of this study.

Boundaries between different channel systems have been drawn where connecting channels (if present) are narrowest. The choice of these boundaries has a limited influence on the analytical results, because the number of channels involved in these choices is relatively small. Similarly, the arbitrary definition of small channels and the arbitrary choice of the confluences has a limited influence on the final measurements, because the amount of choices needed to define the network for the Horton analysis is relatively small. Therefore, the analysis provides reproducible characteristics of the investigated tidal-channel systems.

Results

Network geometry

Horton's hierarchical analysis reveals that the number of channel branches increases logarithmically with decreasing channel order. The curves in Figure 4 have comparable orientations. This indicates that the branching patterns of the investigated tidal-channel systems are comparable, irrespective of the size of the tidal prism or the percentage of inter-tidal flats. In Figure 4 the results of Horton's hierarchical analysis

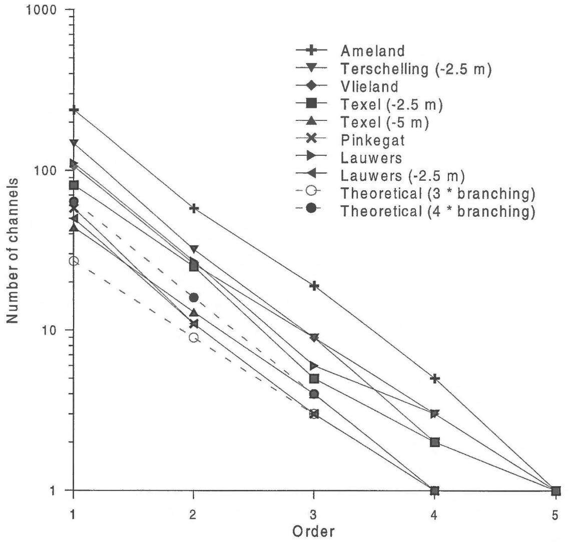

Figure 4. Results of Horton's analysis of tidal-channel systems in the Dutch Wadden Sea. The results of the analysis of theoretical three- and four-times branching networks are indicated as well. See Figure 6 for examples of such networks.

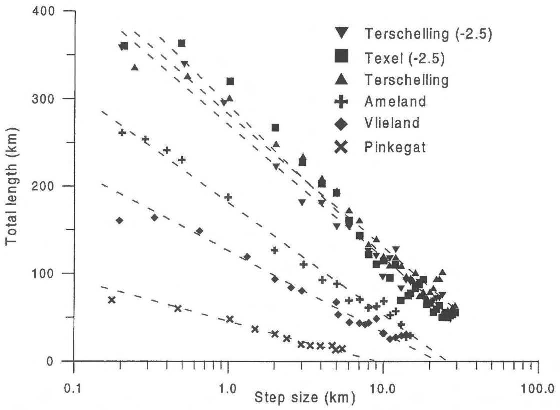

Figure 5. Results of fractal analysis of tidal-channel systems in the Dutch Wadden Sea. The length of the channel-system circumference is plotted logarithmically against the divider length. All channel systems plot as straight lines. The slopes of the lines are related to the channel-system size; large channel systems have steep slopes, small channel systems have gentle slopes. The lengths measured with the smallest divider deviate systematically downward from the straight lines. This indicates the end of channel branching.

of two theoretical three- and four-times branching networks are indicated as well. The curves of the analysed channel networks vary between the orientation of a three-times and a four-times branching network, with a slight preference for the latter. It is noted that especially the higher-order channels tend to deviate. This is mainly due to the method, i.e., the number of high-order channels is low and statistical deviations are likely to dominate the orientation. As a first estimate, the branching pattern of the tidal-channel systems can be regarded as a three- to four-times branching network.

Fractal analysis of the channel-system circumference indicates that its decrease is linear with a logarithmic increase in divider length (Figure 5). Note that the *Y*-axis shows the total length, and not the logarithm of the total length, as opposed to the usual plots of fractal analysis results (Mandelbrot 1967, 1982). With a greater divider length, the amount of detail (small channels), that is included in the measurement, drops rapidly. The slopes of the lines are related to the sizes of the channel systems. Large channel systems have a large channel-system circumference, and therefore a steep slope. This relation is further investigated below.

The linear increase of measured channel-system circumference with the logarithmic increase in divider length is also observed in the fractal analysis of a theoretical network (Figure 6(a)). The length of the branches of the theoretical network decreases logarithmically (Figure 6(b)). Such a network has fractal properties. The similarity in results of the fractal analysis of the theoretical network and the natural tidal-channel systems proves that the theoretical network is a good model for tidal-channel systems. This implies that the tidal-channel systems themselves are fractals.

The measured length of the channel-system circumference does not increase further below the scale of 500 m This implies that channel branching ends below the 500 m scale. This is also observed on the detailed maps and on aerial photographs.

The combined results of both analyses indicate that all investigated tidal-channel systems have a similar fractal geometry, regardless of their size. Tidal-channel geometry is comparable with a three- to four-times branching network of which the branch length decreases logarithmically.

Size dependency

Several well-known empirical relations exist between the tidal prism of tidal-inlet systems and other parameters, see for example O'Brien (1931), Dean & Walton (1975), Dieckman et al. (1988), Hume & Herdendorf (1990), Sha (1989), and Van der Spek (1995). These relationships are also valid for the investigated systems (Louters & Gerritsen 1994). Because channel branching does not continue below the 500 m scale, the total length measured with the smallest

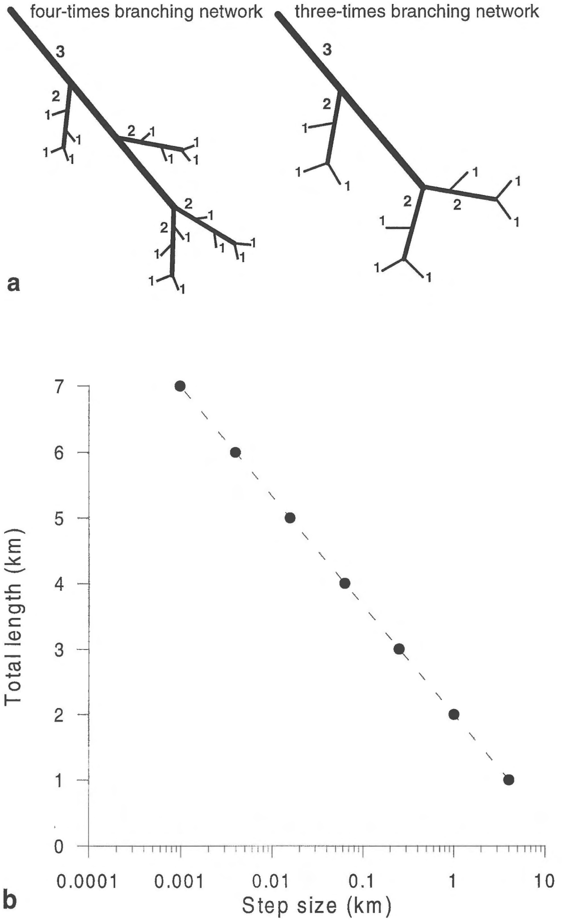

Figure 6. (a) Three- and four-times branching networks, of which the branch lengths decrease logarithmically with the decreasing order (fractal length). These theoretical networks have similar characteristics both in Horton's and in fractal analysis as the natural tidal-channel systems. (b) Results of fractal analysis of four-times branching network. Compare to the results of the fractal analysis in Figure 5.

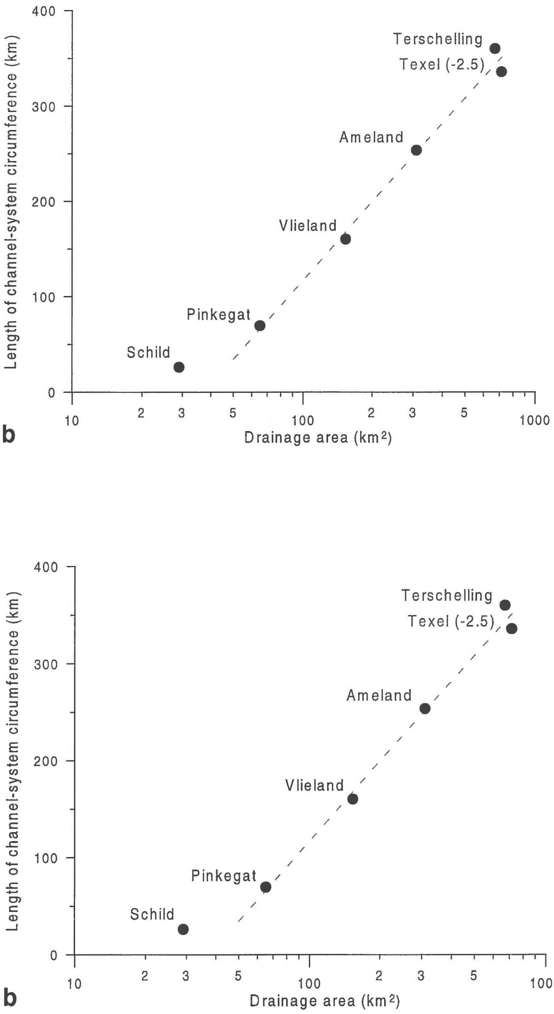

Figure 7. (a) Channel-system circumference length versus the logarithm of the tidal prism of tidal-channel systems. The channel-system circumference length is obtained from the smallest divider length used in the fractal analysis. (b) Channel-system circumference length versus the logarithm of the drainage area of tidal-channel systems.

divider length (ca. 200 m) gives a good approximation of the length of the channel circumference. This allows the determination of a relation between the channel-system circumference and the tidal prism. In Figure 7(a) the channel-system circumference is plotted against the logarithm of the tidal prism. Although the number of data points is limited, a logarithmic relation between channel-system circumference and tidal prism is obvious. A similar relation exists between channel-system circumference and drainage area of the tidal channel systems (Figure 7(b)).

An important implication of these relationships is the minimum size for the equilibrium branching pattern. The straight fit of Figure 7(a) intersects the X-axis at approximately $55 * 10^6$ m^3. This implies that the fractal geometry does not apply to systems with a tidal prism under $55 * 10^6$ m^3.

Discussion

Branching pattern

The analyses reveal similar branching patterns in all investigated tidal-channel systems (Figures 4, 5). The geometry of tidal channel systems is partly random, i.e., the channel lengths and the positions of channels of the same order are not exactly the same. The channel systems can therefore be regarded as statistical self-similar fractals (Voss 1988). The three- and four-times branching networks (Figure 6(a)) are simplifications of this pattern, in which the random component of the branching pattern is missing. However, this model does represent the characteristics of the channel systems that have been analysed.

The fractal dimension of the channel systems is a measure of their surface-filling capacity. Generally, the fractal dimension of drainage networks is given by the relation between the channel lengths and their drainage area (Mandelbrot 1982, Turcotte 1992). However, due to the low gradients in relief and to the complicated flow structure in the Wadden Sea, a differentiation between drainage areas is very subjective, or even impossible. This excludes the application of this method. Therefore, a different option is used, viz. the fractal dimensions of the three- and four-times branching networks. The application of the box-counting method (Turcotte 1992) on the three- and four-times branching networks of Figure 6 gives fractal dimensions of 1.43 and 1.35, respectively. This fractal dimension is a measure for the branchiness of the channel pattern.

From Figure 7 it appears that channel systems with a tidal prism smaller than $55 * 10^6$ m^3 deviate considerably. This suggests that such systems are unstable. Indeed, all but one channel system of the Dutch Wadden Sea have tidal prisms in excess of $55 * 10^6$ m^3. The one exception is Schild Inlet (Table 1), which is known to be decreasing in size since at least 1800, but likely since 1650 (Oost 1995). This unstable inlet is expected to disappear in the next century.

The similarity of the branching patterns (Figures 4, 5) and the logarithmic relation between tidal prisms, drainage lengths and channel-system circumference lengths (Figure 7) result from the evolution of the tidal-channel systems. After initial formation of a channel system, the morphology will increasingly influence the flow pattern. Thus, during evolution, channel-system dynamics become more deterministic, possibly to the point of total-self organization. The deterministic nature of the channel network cannot be inferred from the observed fractal pattern of the tidal-channel systems. However, the geometry is independent of the relative amount of tidal flats, of the size of the drainage basin, and of the tidal amplitude, and this strongly suggests the self-organised nature (Rigon et al. 1994) of the tidal channel systems.

In the current study only the sandy channel systems of the Wadden Sea have been investigated. A non-quantitative comparison with muddy tidal systems (cf. Van Straaten 1964, Reineck 1995), suggests that the lower size boundary for branches in muddy systems is smaller (~10 m). This is thought to be related to the high erosion threshold and high pick-up velocity of the cohesive mud particles and the lower permeability of mud deposits. It suggests that the morphology of tidal-channel systems may differ with different grain-sizes. Other parameters, like wind influence, strong biological control (vegetation: salt marshes and mangroves) and much larger or smaller tidal amplitudes, may have a similar influence on the channel-system geometry.

The similarity in network geometry of tidal-channel systems and river networks has been mentioned in the introduction. River networks have a comparable stochastic or deterministic nature (Shreve 1969, Smart 1979, Stark 1991, Maritan et al. 1996). However, there is a fundamental difference between tidal-channel systems and river networks. In river networks water carries sediment downslope only. Therefore, sedimentation is restricted to the lower stream domains. In tidal basins the flood current carries sediment upslope, enabling sedimentation throughout the drainage basin. Therefore, the position of the smallest, first-order, tidal channels is not merely determined by erosion effects of the draining currents, but also by sedimentation.

Processes

The branching pattern of tidal-channel systems is the result of the processes that control their morphological development. From the similarity of all systems, and from their fractal character it follows that the ruling processes are similar on all scales. These processes do not depend on the relative amount of inter-tidal flats, the tidal amplitude, or the drainage basin size (above a certain limit). Channel formation and maintenance are, at all scales, governed by the balance between erosion and deposition. This balance is controlled by the hydrodynamics, being the resultant of interactions of the tides with the morphology. The resulting morphodynamic feedbacks (Cowell & Thom 1994) are present throughout tidal-channel systems.

In first instance, the geometry of tidal-channels systems can be thought of as an ebb-related drainage feature. This is supported by the observation that maximal tidal-flow velocities in the tidal channels occur during ebb (Van Straaten 1954). The shaping capacity of the ebb currents is therefore larger than that of the flood currents. Channel maintenance requires a certain flow velocity. Channels cannot be maintained if the equilibrium shear-stress needed for their maintenance is larger than the maximum bottom shear-stress. This holds for tidal channels (Van Bendegom 1949), as well as for tidal inlets (Escoffier 1940, Van der Kreeke 1990, 1992). The flow capacity depends on the tidal prism, which in turn is related to the tidal amplitude and drainage area. It implies that each channel necessarily has to have a certain drainage area, and this limits the amount of channels that can be maintained in (a part of) the drainage basin. The distribution of these channels should be such that it enables an optimum drainage of the basin. Furthermore, small channels have a large sediment-water interface relative to big channels. Thus, maximum bottom shear-stress decreases with decreasing channel dimensions. This imposes a threshold, below which the exerted bottom shear-stress is not able to maintain the channel.

The simple-optimum ebb-related drainage pattern is modified by a number of flood-related processes:

1. The asymmetry of the tide results in the flood-dominated accretion of tidal flats (Van Straaten & Kuenen 1958, Postma 1961).
2. The formation of channel levees results from the drop in flood-current velocity when the currents spread over the tidal flats, causing a decrease in shear stress. This limits the sand-transporting capacity of the flood current, resulting in deposition on the levees. However, with increasing height of the levees current velocities above the levees increase, until a net nondeposition equilibrium is established. The ebb flow on the tidal flats is influenced by the presence of the levees. Ebb current velocities have to be high enough to erode the levees, resulting in localised occurrence of (inter-tidal) gullies, which are relatively unstable (Reineck 1995).
3. Furthermore, the eroding capacity of the ebb flow is somewhat reduced by the groundwater reflux, which occurs in inter-tidal flat areas.

All these ebb- and flood-related processes limit the erosion effects of the ebb current and favour deposition on the tidal flats and their boundaries. This determines the end of channel branching below the 500 m scale, and also restricts the number of first-order channels.

Another control on tidal-channel distribution and geometry is the formation of ebb and flood chutes and meandering. Both effects result from inertia of the flow (Van Veen 1950; Van Straaten 1964).

Behaviour-oriented approach

The close interaction between the above-mentioned processes strongly complicates prediction of the morphological development of tidal-channel systems. In combination with the complex self-organisation (Cowell & Thom 1994), the partly random nature of the system's development makes process-based modelling difficult. Understanding tidal-channel systems in terms of 'first-principles' (Rankey & Watney 1996), e.g., straight-forward deposition and erosion, seems inadequate. The relative importance of the different processes changes with, and due to, the evolution of each part of the system. This implies that process-based models necessarily need more detail when long-term predictions are needed.

A behaviour-oriented approach is proposed instead. This approach focuses on equilibrium conditions and the feedbacks maintaining the equilibrium. The observed network geometry is thus regarded as the equilibrium configuration for flooding and draining of the tidal basins in the sandy parts of the Dutch Wadden Sea. This is a dynamic equilibrium; within constant boundary conditions the system changes continuously. This is comparable to the continuous meandering of a river with constant flow (Leopold et al. 1964, Stølum 1996). The similarity in branching between all investigated systems indicates that there is only one dynamic equilibrium configuration.

The geometry of each tidal-channel system fluctuates around its equilibrium. On all scales, deviations from the ideal branching network will result in the re-establishment of the stable configuration, either by silting-up of superfluous channels, or by the development of new channels (Figure 8). These processes are partly stochastic, because the initial position of a channel is random, and partly deterministic, e.g., the presence of a channel excludes the nearby presence of other channels. Branching does not continue below the 500 m scale. The relationships between channel-system circumference, tidal prism and drainage area can be regarded as dynamic-equilibrium relations as well. Application of the equilibrium morphology of the tidal-channel system thus allows modelling of its morphology and its evolution.

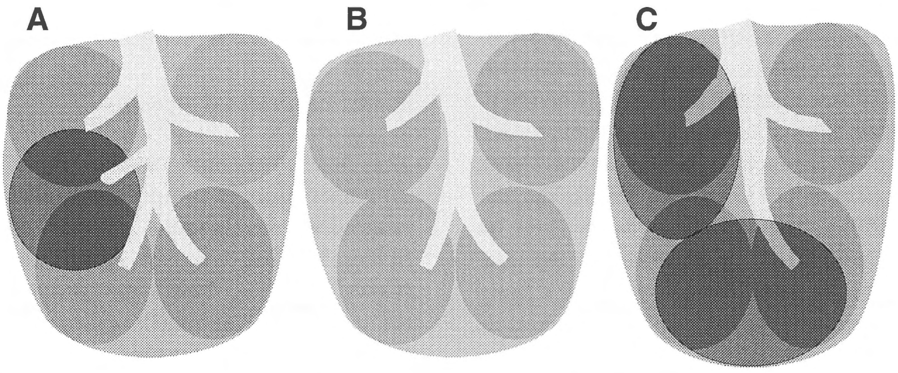

Figure 8. Maintenance of the equilibrium configuration as it occurs on all scales in tidal channel systems. In this case the four-times branching network is regarded as the equilibrium configuration. (A) Unstable configuration, the superfluous tidal channel will silt up and vanish. (B) Stable equilibrium configuration. (C) Unstable configuration, a new channel will be formed by erosion.

Predictions

Potentially, the fractal-network model may improve behaviour-oriented modelling considerably. As yet, mathematical modelling has not been attempted. Below, a first inventory of possible applications in the modern-day and fossil environment is given.

Most modern-day tidal-channel systems are influenced by human interventions, like the dredging of new navigation channels. The fractal-network model indicates that dredging of a new channel leads to silting up of the channel or of a near-by natural channel. By contrast, the damming of natural channels will lead to a new fractal-network configuration of the channel system (with similar branching characteristics) in the remaining tidal basin. These adaptations of channel systems may be modelled more accurately when the fractal-network approach is incorporated in existing behaviour-oriented models (Stive et al. 1996, Buijsman 1997).

Often, fossil channel systems are only partially mapped. The fractal-network approach allows stochastic reconstructions of the unknown parts (Archer & Greb 1995). In a simple approach, the dimensions of fossil tidal channels (in outcrops, seismic reflection data, or cores) can be used to derive their tidal volumes (Van Bendegom 1949, Sha 1989, Van der Spek 1995). This tidal volume in turn relates to a minimum amount of lower-order tidal channels. With this information it is possible to reconstruct a stochastic tidal-channel system (Figure 9). More sophisticated models may incorporate detailed observations, such as the positions of channel fills, and the

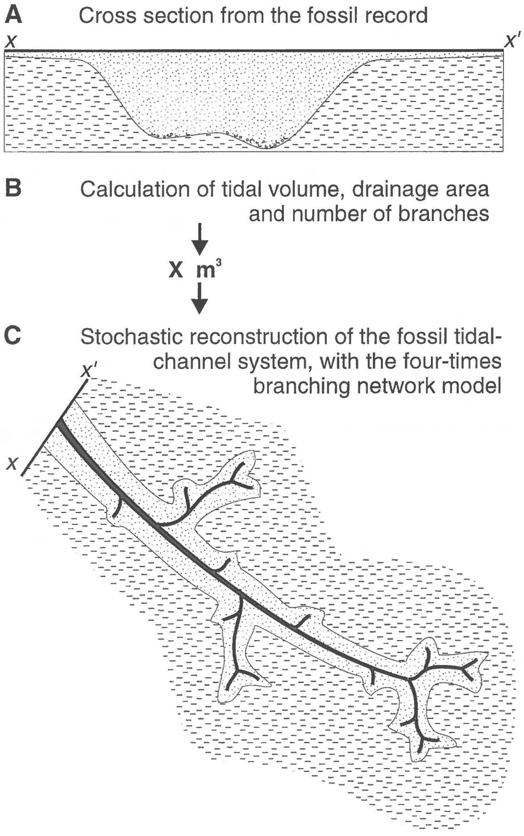

Figure 9. Reconstruction of a fossil tidal-channel system on the basis of limited observations. The ideal network and its size relation with the tidal prism enable predictions of the number and size of tidal channels.

channel lengths, to allow more realistic reconstructions.

The development of fossil tidal-channel systems through time can be considered as well. For example, consider a situation where the tidal volume decreases during a relative rise in sea level. The decrease in tidal volume leads to a reduction of the number and the size of channels, which leaves a clear sedimentary record. Such a change can be modelled stochastically using the three- to four-times branching network, and its size relation with the tidal prism.

Conclusions

Horton's hierarchical analysis and the fractal analysis are simple methods to quantitatively compare tidal-channel systems. The analyses show that all investigated tidal-channel systems have a similar branching pattern. They have the same characteristics as three- to four-times branching networks, with logarithmically decreasing branch lengths. Branching does not continue below the 500 m scale. The channel-system-circumference length derived from the fractal analysis, is correlated to the logarithm of the tidal prism, as well as to the logarithm of the drainage area. The empirical relation implies that channel systems with a tidal prism less than $55 * 10^6$ m^3 are probably unstable. This is supported by the decline of the Schild inlet that has a tidal prism of $31 * 10^6$ m^3.

The independence of the channel-system geometry of size, relative amount of tidal flats and tidal amplitude, suggests that tidal-channel systems are strongly self-organising. Furthermore, the system dynamics are partly random. In first instance, channel networks may be regarded as ebb-driven drainage features. However, a number of hydrodynamic processes, with strong morphological feedbacks, influence this pattern.

The feedbacks between morphology and hydrodynamics result in complex interactions and strong variations of sedimentation and erosion through space and time. This hampers the application of process-based models in the tidal environment. Regarding the three- to four-times branching network as an equilibrium morphology for tidal-channel systems, may improve behaviour-oriented modelling. Furthermore, it allows more reliable stochastic predictions of the distribution of fossil tidal-channel deposits.

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