

## Application of a mechanical model to the Northern Apennines, with special reference to the effect of sea level changes

Leon van den Berg

*Department of Geology, Institute of Earth Sciences Utrecht. Present address: E. Vredelaan 61, 3584 ZB Utrecht, The Netherlands*

Received 28 February 1989; accepted in revised form 23 May 1989.

**Key words:** thrust mechanics, sea level, gravity tectonics, Northern Apennines, melanges

### Abstract

The mechanics and geometry of thin-skinned tectonic wedges are a function of the parameters top and basal slope, strength of wedge and basal layer, pore-fluid pressure and sea level. An analysis of a mechanical model, that takes these parameters into account, shows that alternating submarine and subaerial conditions can have important consequences for the stability of such a wedge.

The model is applied to the Northern Apennines. It can explain multiple phases of gravitational sliding in the Ligurian scaly clay melanges. The contrasting style of deformation of different tectonic units, and temporary suppression of synsedimentary thrusting in the Romagnan sequences can also be explained by this model.

It is shown that gravity spreading, induced by brittle-ductile transition, provides a mechanism which can explain the metamorphic gap at the tectonic boundary between the doubled Tuscan sequences in the Alpi Apuane, as well as the simultaneous spreading at the rear and the shortening at the front of the Northern Apennine foldbelt.

### Introduction

In their well-known publication, Hubbert & Rubey (1959) extended Terzaghi's (1943) soil mechanical rules to geologic conditions, and demonstrated that high pore-fluid pressures can facilitate low angle thrusting. They also showed that water-saturated subaerial blocks can slide from slopes at an angle smaller than for submarine blocks, by a factor of  $(\rho - \rho_w)/\rho \approx 0.6$  (where  $\rho$  is the density of the rock and  $\rho_w$  the density of water). This effect is caused by the down-slope hydraulic gradient, which is absent under submarine conditions (see also Lambe & Whitman, 1979). A consequence is that subaerial blocks cannot slide far into the water if the basal slope remains constant (Chapman, 1979). It is in-

teresting to analyse quantitatively, with an accurate model, to what extent a thrusting system will change if it emerges above sea level.

Many authors established models to describe the mechanics and geometry of thrust systems, obtaining different results, due to varying assumptions about the mechanical properties (Platt, 1986). Davis et al. (1983), Dahlen et al. (1984) and Dahlen (1984) worked out a rather realistic 'bulldozer' model, as earlier outlined by Chapple (1978), to describe the mechanics and geometry of a wedge-shaped fold- and thrust-belt that is underlain by a weak basal layer and is being deformed under high pore-fluid pressure. Assuming deformational stresses, they established a relation between top and basal slope which determines whether short-

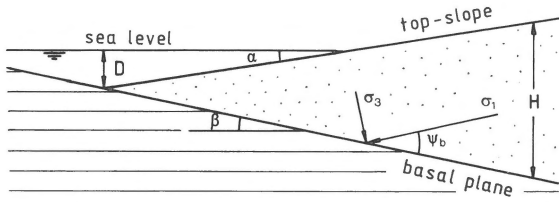


Fig. 1. Diagram of a wedge with top slope  $\alpha$ , basal slope  $\beta$ , active principle stresses  $\sigma_1$  and  $\sigma_3$ , depth of toe  $D$  and thickness of wedge  $H$ .  $\psi_b (= \alpha + \beta)$  is the angle between  $\sigma_1$  and the basal plane.

ening or spreading will occur, or whether only sliding at the basal plane is effective. They concluded (Davis et al., 1983) that a submarine wedge can have a steeper top slope than its subaerial counterpart.

In this paper the effect of sea level changes on shortening and spreading wedges will be analysed quantitatively. Furthermore, this model will be used to explain some tectonic features within the Northern Apennines.

### Mechanical model of thin-skinned tectonic wedges

The geometry of a wedge can be described by a top slope with dip  $\alpha$  and a basal slope with dip  $\beta$  (Fig. 1). In a thin-skinned wedge that is underlain by a relatively weak basal layer, we can assume brittle Mohr-Coulomb behaviour. The mechanical behaviour can be described by the strength of the wedge material with an internal friction coefficient  $\mu$  and a pore-pressure ratio  $\lambda$ , underlain by basal plane material with an internal friction coefficient  $\mu_b$  and a pore-pressure ratio  $\lambda_b$ .

Within an internally deforming wedge, the two-dimensional active principle stresses  $\sigma_1$  and  $\sigma_3$  define a Mohr-circle diagram (Fig. 2). If the internal friction of the basal plane is lower than the internal friction within the wedge, then the Mohr-circle has two points of intersection with the line of failure of the basal plane material (Fig. 2). These two points of intersection yield the angles  $\psi_{bc}$  and  $\psi_{bs}$ , which are the angles between  $\sigma_1$  and the basal plane for horizontal shortening (compression) and spreading, respectively. If a wedge is internally deforming

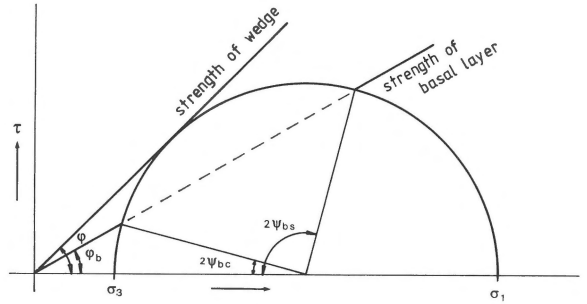


Fig. 2. Mohr-circle diagram of stresses within a wedge with a weak basal layer, at the condition of failure.  $\phi$  and  $\phi_b$  are the angles of internal friction within the wedge and at the basal plane, respectively. Note the two points of intersection at the Mohr-circle, with the two angles of orientation of the stress field  $\psi_{bc}$  and  $\psi_{bs}$  for compression and spreading, respectively.

under submarine conditions, top slope  $\alpha$  can be explicitly defined by

$$\alpha = \frac{(1 - \lambda_b)\mu_b + (1 - \rho_w/\rho) \beta}{(1 - \rho_w/\rho) + (1 - \lambda)K_{c/s}} - \beta \quad (1)$$

where  $K_c$  and  $K_s$  are two constants that correspond to horizontal shortening (compression) and spreading, respectively (see appendix). As a consequence, for given mechanical parameters and basal slope  $\beta$ , two values for top slope  $\alpha$  exist: a maximum value at which a wedge deforms internally by spreading, and a minimum value at which a wedge deforms internally by shortening. If a wedge has a top slope which is between these two values, the differential stress ( $\sigma_1 - \sigma_3$ ) within the wedge is not high enough to cause internal deformation, and the wedge slides at its basal plane only.

We will now further analyse some special cases by inserting parameters, relating to realistic situations. Fig. 3 is a Mohr-circle diagram of an internally deforming wedge, having the following parameters: for the wedge material we adopt the empirical value for  $\phi (= \text{atn } \mu)$  of  $40.4^\circ$  (Byerlee, 1978); for the weak basal plane material we use a value  $\phi_b (= \text{atn } \mu_b)$  of  $8^\circ$ . Such a low value is typical for many micaceous materials, as has empirically been found by Summers & Byerlee (1977) and Shimamoto & Logan (1981) for smectites, by Borowicka (1965) and Raj (1981) for natural clays, by Van den Berg (1987) for scaly clays, and by Nascimato (1981) for wet white micas. For the pore-

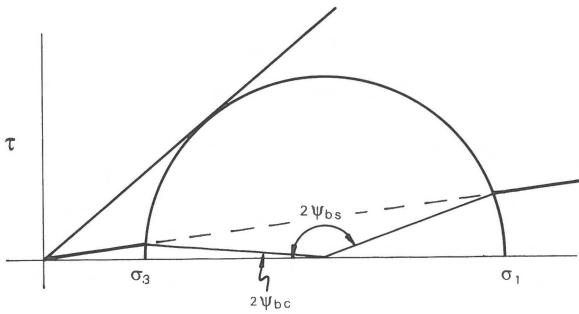


Fig. 3. Mohr-circle failure diagram of a wedge with  $\varphi = 40.4^\circ$  and  $\varphi_b = 8^\circ$ . Note that the orientation of  $\sigma_1$  is nearly parallel to the basal plane for shortening, or nearly perpendicular to the basal plane for spreading.

pressure ratio  $\lambda$  a value of 0.67 is selected, conform to Davis et al. (1983). Figure 3 illustrates that  $\psi_{bc}$  is very low and  $\sigma_1$  will be oriented nearly parallel to the basal plane in the case of shortening, and  $\psi_{bs}$  will be large, and  $\sigma_1$  will be oriented nearly perpendicular to the basal plane in the case of spreading.

### Submarine versus subaerial conditions

Equation (1) allows us to draw graphs which, for given mechanical parameters, show the relation between  $\beta$  and the two possible values of  $\alpha$ . The two possible values for  $\alpha$  define an area where a wedge is internally stable. In Fig. 4 these graphs are shown for wedges under submarine and subaerial conditions, having the same mechanical parameters as used in Fig. 3.

From Fig. 4 we can deduce the consequences when a submarine wedge becomes subaerial. With a constant slope ( $\beta$ ) of the basal plane a spreading wedge, labelled 1, will spread until it reaches a position labelled 1'. A shortening wedge, labelled 2, however, will become internally stable. A shortening wedge, labelled 3, will shorten until it reaches a position labelled 3'. A shortening wedge, labelled 4, will become a spreading wedge.

From Fig. 4 we can also deduce the consequences when a subaerial wedge becomes submarine. The spreading wedge labelled 1' will become internally stable, a shortening wedge labelled 2' will shorten until it reaches the position labelled 2. A

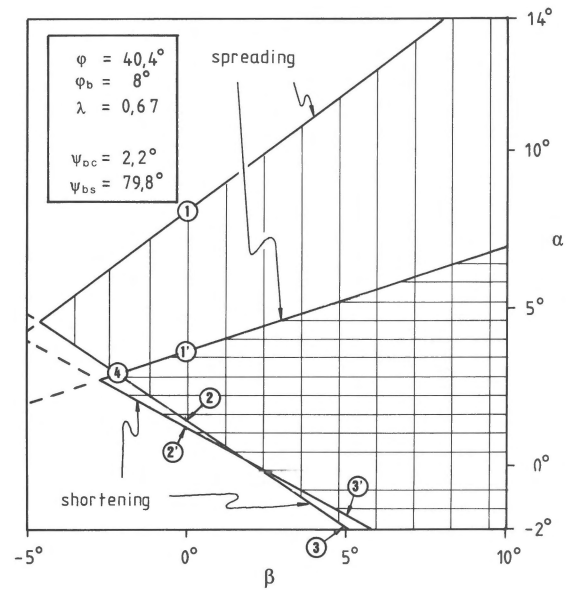


Fig. 4. Stability fields for wedges under submarine (vertically hachured) and subaerial conditions (horizontally hachured), based on the relations between  $\alpha$  and  $\beta$ . Wedges within the hachured zones are internal stable and will slide at the basal plane only. Wedges at the edge of a stability field will deform internally and at the base. Indicated points are discussed in the text.

spreading wedge, labelled 4, will become a shortening wedge.

We can conclude that spreading wedges can have a steeper top slope when they are submarine, and that they will be forced to spread if they become subaerial. For shortening wedges, the consequences of a change from submarine to subaerial conditions and vice versa will depend on the basal slope.

It is important to realize that if the top slope of a shortening wedge increases, e.g. due to shortening or sedimentation upon the wedge, the wedge should become internally stable.

To show the effect of a higher pore-fluid pressure, in Fig. 5  $\lambda$  is increased to 0.80, while the other mechanical parameters are kept as in Fig. 4. It shows that under these circumstances, for all values of  $\beta$ ,  $\alpha$  is closer to zero.

The relation between  $\alpha$  and  $\beta$  for a wedge in which the mechanical behaviour is controlled by one weak material only, for instance a clay melange where  $\varphi = \varphi_b = 8^\circ$ , is shown in Fig. 6. We see that for each value of  $\beta$  only one value for  $\alpha$  exists, and

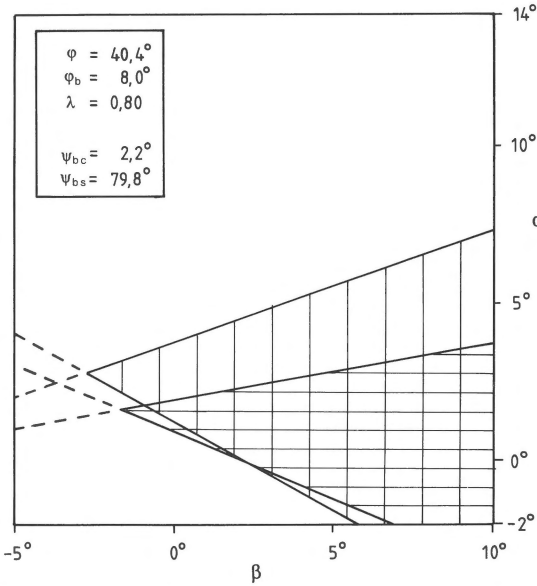


Fig. 5. Stability field for wedges, having the same mechanical parameters and conditions as Fig. 4, but a higher pore-fluid pressure. Relations are essentially the same, but  $\alpha$  is closer to zero for all values of  $\beta$ .

that the areas where a wedge is internally stable have disappeared. This is the logical consequence of the absence of a difference in internal friction between the wedge and the basal layer (see Fig. 2). Deformation takes place by faults which are parallel to the top-slope ( $\psi_{bc} = \psi_{bs} = 41^\circ$ ).

#### *The effect of a gradual change in sea level*

Up till now we described abrupt changes from submarine into subaerial conditions and vice versa. Let us now envisage the more realistic situation in which the sea level changes gradually, resulting in wedges which are partly submarine, partly subaerial. In Fig. 7 these situations are sketched for the wedges 1-1', 2-2', 3-3', 4 and 5-5', which were labelled in Figs. 4 and 6. It shows that if an internally deforming wedge is partly submarine, partly subaerial, the top-slope will vary along its cross-section (1-1', 2-2', 3-3', 5-5') or change from spreading to shortening conditions (4).

It is interesting to examine to what extent wedges will spread or shorten as a result of changes in sea

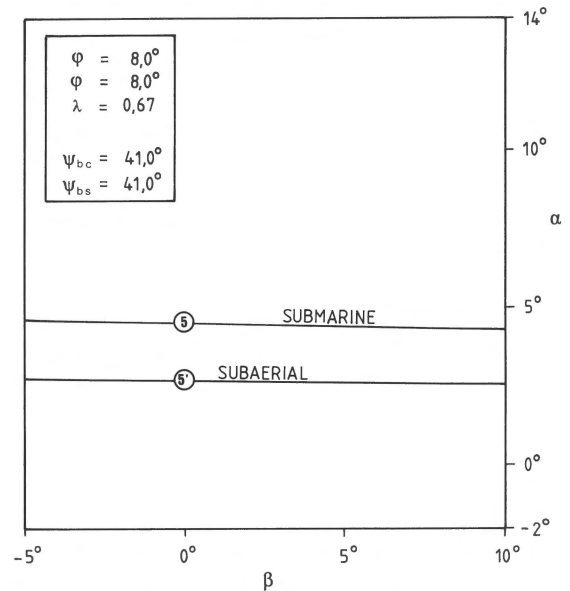


Fig. 6. Relations between  $\alpha$  and  $\beta$  for wedges which are composed of clay only. The stability fields have disappeared completely, and  $\alpha$  is almost independent of  $\beta$ . Note that the angle between  $\sigma_1$  and the basal plane is  $41^\circ$ , thus deformation will take place parallel to the top slope.

level. To evaluate this, we will analyse the specific situations, labelled in Figs. 4 and 6. We will give them an initial arbitrary length of 50 km when they are fully under water. We will then lower sea level, and check to what extent the wedges will spread or shorten. The cross-sectional area of the wedges is kept constant during deformation. In Fig. 8 the length of the wedges is plotted versus the depth  $D$  of the toe. It shows that the effect of a change in sea level is strongest for the spreading wedges, when the toe is near sea level. Fig. 9 is mainly constructed as Fig. 8, but shows more detail. The initial lengths of the wedges are taken as 50 km when the depth of the toe is 500 m. It shows that a change in sea level of 100 m can have an effect for horizontal spreading or shortening in the order of 2-3 km.

In the following sections, some of the structural aspects of the Northern Apennines are interpreted by the model here developed.

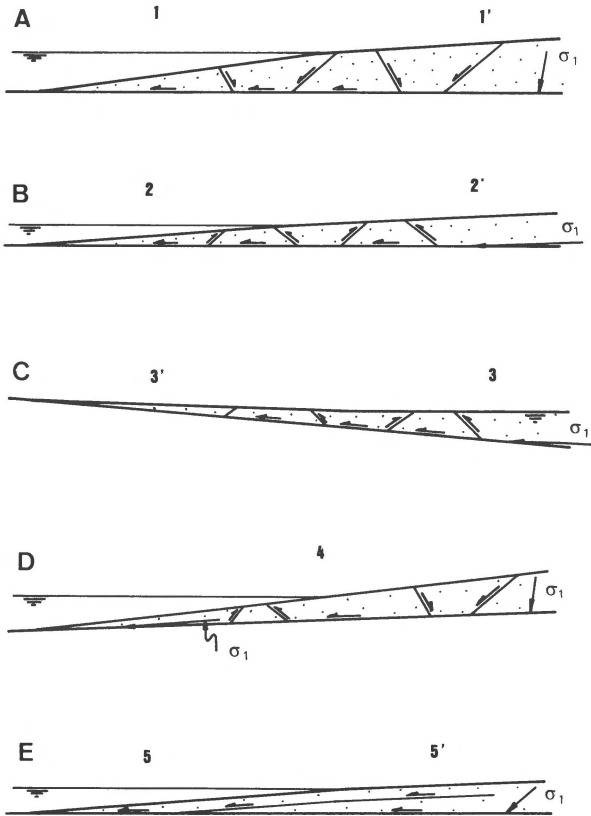


Fig. 7. Diagrams, illustrating wedges which are partly submarine, partly subaerial. Numbers above the wedges correspond to the labels, used in Figs. 4 and 6. Arrows indicate orientation of  $\sigma_1$ . Fig. 7a: spreading wedge 1-1'. The top slope is lower under subaerial conditions. Fig. 7b: shortening wedge 2-2'. The top slope is lower under subaerial conditions. Fig. 7c: shortening wedge 3-3'. In contrast to wedge 2-2', the top slope is steeper under subaerial conditions. Fig. 7d: wedge 4. The top slope is constant along the cross section; the subaerial part is spreading and the submarine part is shortening. Fig. 7e: wedge 5-5', which is subjected to gravitational sliding. Faults are parallel to the top slope, which is lower under subaerial conditions.

### Application to the Northern Apennines

#### *Phases of tectonic advance and sea level changes*

Fig. 10a shows the main tectonic characteristics of the external units of the Northern Apennines. The Ligurian units were thrust over the Tuscan and Romagnan units synsedimentary, and the advance appears as a 'tectonic transgression' (Merla, 1952; De Jager, 1979). The interfingering of the Ligurian scaly clay melanges into the Romagnan flysches

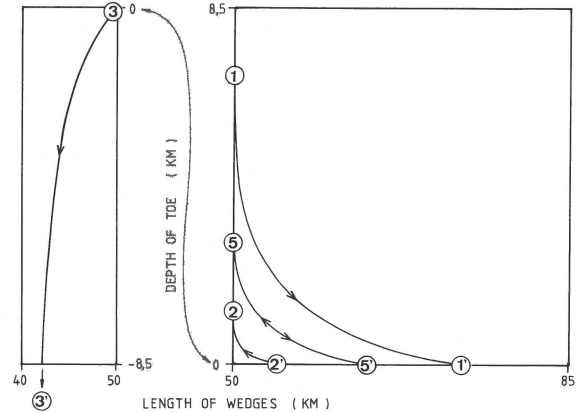


Fig. 8. Change in length of some wedges, labelled in Figs. 4-6, as a function of the depth of the toe below sea level. Cross-sectional area of the wedges. Left part of the figure shows the height of toe above sea level of wedge 3-3', which has a top slope dipping away from the toe, thus the depth of the toe is negative. Note that the induced senses of deformation, indicated by the arrows, are restricted.

allows us to trace periods of advances of the Ligurian 'tectonic transgression'. According to our model, a positive correlation between sea level drops and phases of advance might be expected, though it is possible that such a correlation is obscured by eutectonic movements.

A phase of eustatic sea level drop in the Middle Serravalian as found by Haq et al. (1987) coincides with a phase of synsedimentary thrusting of Ligurian units over the Romagnan units, and with advances and thrust-tectonic events in other areas (De Jager, 1979; Meulenkamp & Hilgen, 1986). Also the advances of the Ligurian 'tectonic transgression' in the Romagnan region at the end of the Tortonian (De Jager, 1979; Ruggieri, 1970; Ricci Lucchi & D'Onofrio, 1967), during the Messinian (Drooger, 1973) and the Early Pliocene (Ceretti & Colalongo, 1984; Castellarin et al., 1985; Conti, 1987) seem to coincide with eustatic sea level drops (Haq et al., 1987), and gravity sliding and nappe emplacement in other areas (Meulenkamp & Hilgen, 1986).

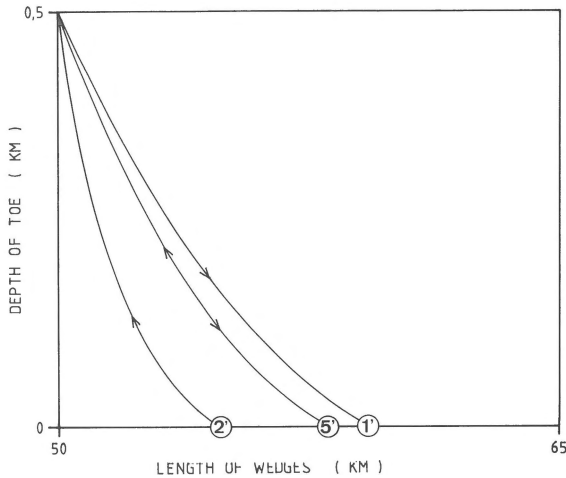


Fig. 9. Change in length of some wedges as a function of the depth of the toe below sea level. The initial length is taken here as 50 km at 500 m depth. Arrows indicate the induced sense of deformation.

#### *Contrast in deformation style between different tectonic units*

In Fig. 10a we can observe an amazing difference in style of deformation between the Tuscan and Romagnan units and the Ligurian units since Tertiary times: The Tuscan and Romagnan units are intensely shortened by folding and thrusting (Koopman, 1983; De Feyter et al., 1990, this volume; Treves, 1984; Ten Haaf en Van Wamel, 1979; Van Wamel & Zwart, 1990, this volume). The Ligurian units, however, seem hardly deformed since the Tertiary: the Ligurian Tertiary molasses are unfolded, and the Ligurian oceanic flysches – although spectacularly folded during earlier tectonic phases (Ten Haaf, 1985; Hsü, 1967; Van Wamel et al., 1985) – appear only slightly deformed since the Ligurian Tertiary molasses were deposited on top of them. The extension of disjointed slabs within the Ligurian hanging wall and the obvious lack of any shortening tectonics characterize the deformation style of the Ligurian thrusts as gravity sliding and/or spreading.

The contrast between the shortening tectonics in the Tuscan and Romagnan units and the gravitational sliding/spreading of the Ligurian units at the same time, may be explained by differences in their mechanical behaviour. The ensialic flysches and

Mesozoic carbonates of the Tuscan and Romagnan units deformed as a wedge of strong beds with a high internal friction, underlain by the weak basal anhydritic Burano formation (Pieri & Mattavelli, 1986), having an almost horizontal top-slope (Van Wamel & Zwart, 1990, this volume), and was therefore subjected to shortening (see Figs. 3, 4 and 5). The external Ligurian units, however, were surrounded by weak scaly clays, which controlled their mechanical behaviour, and were therefore subjected to gravitational sliding (see Figs. 6 and 7).

The fact that the Ligurian units were intensely folded and thrust in earlier tectonic phases, but were solely subjected to gravitational sliding since the Tertiary, might be explained by assuming that initially scaly clays were present only at the stratigraphical bases of the Ligurian flysches (Zanzucchi, 1980; Van Wamel et al., 1985). The Ligurian flysches, underlain by this weak basal layer, were subjected to shortening (see Fig. 4). Due to the intense thrusting, folding and diapirism (see Chapman, 1974; Treves, 1985; Barber et al., 1986) the scaly clays got completely intermixed with the Ligurian flysches and semi-allochthonous molasses, and the Ligurian units started to behave as clay melanges, subjected to gravitational sliding.

#### *Suppression of synsedimentary thrusting in the Romagnan units*

The thrusts within the Romagnan ensialic flysches were synsedimentary (De Jager, 1979; Ten Haaf, 1985; Van Wamel & Zwart, 1990, this volume; De Feyter et al., 1990, this volume) (Fig. 10a). Along the Sillaro line, thrusting of the Ligurian units over the Romagnan units has suppressed movement along the Romagnan thrusts, since the Romagnan thrusts do not intersect the Ligurian thrust. The thrust deformation front of the Romagnan units migrated to the external foothills (Bernini & Clerici, 1983) and the Padan-Adriatic region, where thrusting continued at least up to the Late Pliocene (Castellarin et al., 1985) (see Fig. 10a). In the internal zones subsequent reactivation of the thrusts within the Romagnan units locally caused intersec-

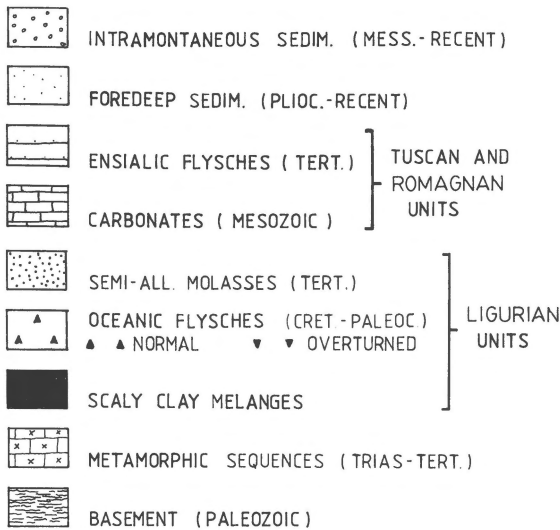
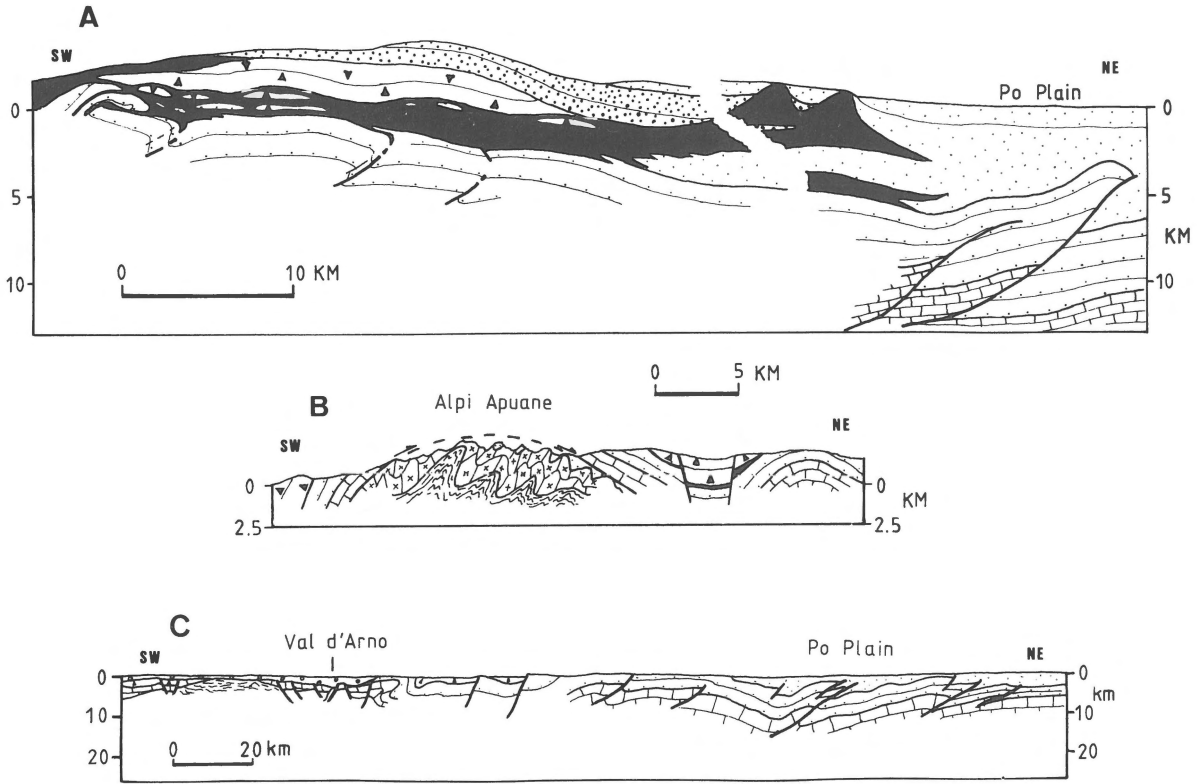


Fig. 10. Sections across the Northern Apennines.

A. Composite cross-section through the external Apennines, near Bologna, based on De Jager, 1979; Ten Haaf, 1985; Pini, 1987; Castellarin et al., 1985 and own data. The translation along the syndedimentary faults in the Tuscan and Romagnan ensialic flysches is suppressed by the 'tectonic transgression' of the Ligurian scaly clay melanges. The Ligurian oceanic flysches, after having been isoclinally 'folded', and covered by molasses, were translated only, almost without any further deformation, in contrast to the footwall.

B. Synthetic cross-section through the Alpi Apuane, modified after Boccaletti & Coli, 1982; Carmignani et al., 1978; Dallan Nardi & Nardi, 1979. Structures are extrapolated along the main fold axis. The Tuscan units are doubled. The metamorphic Tuscan units are directly overlain by the unmetamorphic Tuscan units.

C. Cross section from Livorno to the Po plain, after Boccaletti & Coli (1983). Note the recent thrusting at the front and normal faulting at the internal side of the foldbelt.

tion of these thrusts with the Ligurian thrust, thus this suppression was temporary only.

This suppression may be explained by an increase of the top slope of the Romagnan wedge, caused by the overthrusting of the Ligurian units over the Romagnan wedge, which made the Ro-

magnan wedge internally stable. Also, the tectonic loading of the impermeable Ligurian scaly clay melanges increased the pore-fluid pressure and may have contributed to an internal stable Romagnan wedge (see also Dahlen, 1984). The subsequent reactivation might have been caused by a

later decrease in top slope and pore-fluid pressure, when the main masses of the Ligurian units were (tectonically) eroded.

### *The Alpi Apuane*

The tectonic window in the Alpi Apuane has led to a longstanding problem. In this window, the complete Tuscan sequence, consisting of Mesozoic carbonates and Tertiary ensialic flysches, is doubled (Fig. 10b). The thrust plane is formed by the Triassic Burano anhydrite formation, which completely cuts off the folds and thrusts within the footwall. The hanging wall is essentially unmetamorphosed, but the footwall is regionally metamorphosed in the greenschist facies (Carmignani et al., 1978; Crisci et al., 1975), thus there is a rather large metamorphic gap at the thrust plane. Such a structural setting is in fact very common in many orogens (Platt, 1986). It has been explained by assuming obduction of the metamorphic sequences after an earlier subduction (Dallan Nardi & Nardi, 1979), or local shear heating (Carmignani et al., 1978). Platt (1986) argued that continued underplating could bring sediments at high-P/low-T conditions, and would thicken the wedge. The top slope would become too steep and induce internal spreading, expressed as listric normal faults, and that could explain such a metamorphic gap at many major tectonic contacts.

We will now present an alternative hypothesis to explain the metamorphic gap in the Alpi Apuane as well as in orogenes in general. Within a shortening and thickening wedge which deforms brittly, stratigraphic units will be doubled by thrusting. If, in the internal zones of such a wedge, the brittle-ductile transition would be achieved at the basal plane, the friction at the basal plane would be reduced, and the wedge would become internally stable. A continuing increase of temperature will make the wedge material ductile also, and provided the wedge had a positive top slope, it will spread under its own weight, to obtain a lower top slope (see also Davis et al., 1983). This spreading could take place by listric normal faulting or by reactivation of thrust faults as normal faults, which would

bring unmetamorphic rocks directly upon metamorphic rocks. In this interpretation the thrust plane in the Alpi Apuane is a thrust fault which has been reactivated as a normal fault.

When the internal zones of an orogene are spreading ductily, brittle Mohr-Coulomb behaviour can still be effective at the front of the wedge, associated with shortening. This phenomenon is characteristic for the Northern Apennines (Malinverno & Ryan, 1986; Pieri & Mattavelli, 1986; Boccaletti & Coli, 1983) and might be presently active, since nowadays shortening takes place externally, in the Padan-Adriatic region, whereas spreading occurs more internally, causing intramontaneous graben structures (see Fig. 10c).

The effect of spreading, induced by brittle-ductile transition, would be mainly the same as predicted by Platt (1986): within the orogenic wedge simultaneous shortening at the front and spreading in the internal zones would occur, together with a metamorphic gap at tectonic boundaries.

### **Conclusions**

Our mechanical model can help to interpret the mechanics and geometry of deforming wedges. It can show the effect of changes in the parameters that control the stability of these wedges, such as top and basal slope, sea level, pore-fluid pressure and strength, and is consistent with observed structures in the Northern Apennines.

A change from submarine into subaerial conditions can have important consequences for the stability of a wedge. A 50 km long wedge, in a state of spreading or gravitational sliding, may advance 2–3 km for a sea level drop of 100 m. This effect can explain the positive correlation between eustatic sea level drops and phases of advance of the Ligurian units over the Romagnan units.

The style of deformation can be determined by the mechanical parameters of the deforming material.

The covering of shortening wedges by clay melanges or sediments can make these wedges internally stable, due to increased top slope and pore-fluid pressure. This may have been the mech-

anism which temporarily suppressed synsedimentary thrusting within the Romagnan units.

The metamorphic gap at the Tuscan thrust in the Alpi Apuane, and the simultaneous shortening at the front and spreading at the rear end of the Northern Apennine foldbelt might be caused by the behaviour of the brittle-ductile transition at depth in the deforming wedge. This might also apply to other, comparable orogenes.

### Acknowledgements

I thank Prof. E. Ten Haaf and Dr W.A. van Wamel for their significant contributions to this paper and to my geological education.

### Appendix

In case of a critical state of stress throughout a wedge, according to Davis et al. (1983), the top slope is described by

$$\alpha = \frac{(1 - \lambda_b)\mu_b + (1 - \rho_w/\rho)\beta}{(1 - \rho_w/\rho) + (1 - \lambda)K} - \beta \quad (1a)$$

where  $K$  is a dimensionless quantity that is defined by

$$K = 2H^{-1} \int_0^H \frac{dz}{\csc \varphi \sec 2\psi(z) - 1} \quad (1b)$$

where  $\psi(z)$  is the angle between the basal plane and  $\sigma_1$ , changing with local depth  $z$ , and  $H$  is the local thickness of the wedge. In this model, the basal shear stress  $\tau_b$  for submarine conditions can be described by

$$\tau_b = (\rho - \rho_w)gHa + (1 - \lambda)K\rho gH(\alpha + \beta) \quad (1c)$$

where  $g$  is the acceleration of gravity. The term  $(\rho - \rho_w)gHa$  corresponds to the gravity effect of the sloping top surface, as was formulated by Elliot (1976), and the term  $(1 - \lambda)K\rho gH(\alpha + \beta)$  is the consequence of the resistance of the wedge to deform internally by horizontal shortening. Since  $K$  varies with depth, (1a) does not yield an exact definition of the critical top slope. Dahlen (1984), however, showed that in the case of a noncohesive wedge, the orientations of the principal stresses  $\sigma_1$  and  $\sigma_3$  within a wedge are constant, as defined by

$$\psi_b = \frac{1}{2} \arcsin(\sin \varphi_b' / \sin \varphi) - \frac{1}{2} \varphi_b' \quad (1d)$$

where  $\varphi_b'$  is the effective angle of internal friction. Equation (1d) yields two possible values of  $\psi_b$ , by the two solutions for the arcsin, bifurcating the solution for a critical top slope. We will denote these two solutions by  $\psi_{bc}$  for the small angle (compression)

and  $\psi_{bs}$  for the wide angle (spreading). They replace  $\psi(z)$  in equation (1b), and are independent of local depth  $z$ . Following the derivation as used by Davis et al. (1983), it can be shown that, since (1d) provides two constants,  $K$  also becomes bifurcated into two constants, defined by

$$K_{cs} = \frac{2}{\csc \varphi \sec 2\psi_{bc} - 1} \quad (1e)$$

where  $K_c$  stands for horizontal shortening and  $K_s$  for horizontal spreading. This result is very similar to what in soil mechanics is referred to as the passive and active Rankine state, with  $K_p$  and  $K_a$  for horizontal shortening and spreading respectively (see e.g. Terzaghi, 1943; Lambe & Whitman, 1979; Craig, 1978). Using now (1d) and (1e)  $\alpha$  can be described by

$$\alpha = \frac{(1 - \lambda_b)\mu_b + (1 - \rho_w/\rho)\beta}{(1 - \rho_w/\rho) + (1 - \lambda)K_{cs}} - \beta$$

The subaerial condition can be found by setting  $\rho_w$  to zero.

### References

- Barber, A.J., S. Tjokrosapoetro & T.R. Charlton 1986 Mud volcanoes, shale diapirs, wrench faults, and melanges in accretionary complexes. Eastern Indonesia – Am. Assoc. Pet. Geol. Bull. 70: 1729–1741
- Bernini, M. & A. Clerici 1983 Individuazione di un campo di sforzi compressivo in alcuni affioramenti del Pleistocene continentale del margine Appenninico presso Collecchio (Parma) – Soc. Geol. Ital., Boll. 9: 369–384
- Boccaletti, M. & M. Coli 1983 La tettonica della Toscana: assetto ed evoluzione – Mem. Soc. Geol. Ital., 25: 51–62
- Boccaletti, M. & M. Coli 1982 Carta strutturale dell'Appennino settentrionale-Consiglio Nazionale delle Ricerche, Florence
- Borowicka, H. 1965 The influence of the colloidal content on the shear strength of clay – 6th Int. Conf. Soil. Mech. Found. Eng. Montreal: 175–178
- Byerlee, J. 1978 Friction of rocks – Pure Appl. Geoph. 116: 615–626
- Carmignani, L., G. Giglia & R. Kligfield 1978 Structural evolution of the Apuane Alps: an example of continental margin deformation in the Northern Apennines, Italy – J. Geol. 86: 487–504
- Castellarin, A., C. Eva, G. Giglia & G.B. Vai 1985 Analisi strutturale del fronte Appenninico Padano – Giorn. Geol. 47 (1–2): 47–75
- Ceretti, E. & M.E. Colalongo 1984 Alloctonia nell'area Padano-Adriatica: il caotico pliocenico della Val Sellustra – Giorn. Geol. 46 (2): 113–126
- Chapman, R.E. 1979 Mechanics of unlubricated sliding – Geol. Soc. Am. Bull. 90: 19–28
- Chapman, R.E. 1984 Clay diapirism and overthrust faulting – Geol. Soc. Am. Bull. 85: 1597–1602
- Chapple, W.A. 1978 Mechanics of thin-skinned fold-and-thrust belts – Geol. Soc. Am. Bull. 89: 1189–1198

- Conti, S. 1987 Studio geologico della geotraversa Grosseto-Val Marecchia tra le valli del Savio e del Foglia (Appennino Romagnolo-Marchiano) – Ph. D. thesis Univ. Bologna Modena: 125 pp
- Craig, R.A. 1978 Soil mechanics – Van Nostrand Reinhold (New York): 318 pp
- Crisci, G.M., L. Leoni & A. Sbrana 1975 La formazione dei marmi delle Alpi Apuane (Toscana): Studio petrografico, mineralogico e chimico – Atti Soc. Tosc. Sc. Nat. Mem. 82 (A): 194–236
- Dahlen, F.A. 1984 Noncohesive critical Coulomb wedges: an exact solution – J. Geoph. Res. 89 (B12): 10125–10133
- Dahlen, F.A., J. Suppe & D. Davis 1984 Mechanics of fold-and-thrust belts and accretionary wedges: cohesive Coulomb theory – J. Geoph. Res. 89 (B12): 10087–10101
- Dallan Nardi, L. & R. Nardi 1979 Il quadro paleotettonico dell'Appennino settentrionale: un'ipotesi alternativa – Atti Soc. Tosc. Sci. Nat. 85: 289–297
- Davis, D., J. Suppe & F.A. Dahlen 1983 Mechanics of fold-and-thrust-belts and accretionary wedges – J. Geoph. Res. 88 (B2): 1153–1172
- De Feyter, A.J., N. Molenaar, G. Piali, M. Menichetti & F. Veneri Palaeotectonic significance of gravity displacement structures in the Miocene turbidite series of the M. Pollo Syncline (Umbro-Marchean Apennines, Italy). Geol. Mijnbouw 69: 69–86
- De Jager, J. 1979 The relation between tectonics and sedimentation along the 'Sillaro line' – Geol. Ultrajectina 19: 98 pp
- Drooger, C.W. 1973 The Messinian events in the Mediterranean. A review. In: C.W. Drooger (ed.): Messinian events in the Mediterranean. North Holland Publ. Company, Amsterdam: 263–272
- Elliot, D. 1976 The motion of thrust sheets – J. Geoph. Res. 81 (5): 949–963
- Haq, B.U., J. Hardenbol & P.R. Vail 1987 Chronology of fluctuating sea levels since the Triassic – Science 235: 1156–1167
- Hsü, J.K. 1967 Origin of large overturned slabs of Apennines – Am. Assoc. Pet. Geol. Bull. 51: 65–72
- Hubbert, M.K. & W.W. Rubey 1959 Role of fluid pressure in mechanics of overthrust faulting – Geol. Soc. Am. Bull. 70: 115–166
- Koopman, A. 1983 Detachment tectonics in the central Apennines, Italy – Geol. Ultrajectina 30: 155 pp
- Lambe, T.W. & R.V. Whitman 1979 Soil mechanics, SI version – John Wiley (New York): 553 pp
- Malinverno, A. & W.B.F. Ryan 1986 Extension in the Tyrrhenian sea and shortening in the Apennines as result of arc migration driven by sinking of the lithosphere – Tectonics 5 (2): 227–245
- Merla, G. 1952 Geologica dell'Appennino settentrionale – Soc. Geol. Ital., Boll. 70: 95–382
- Meulenkamp, J.E. & F.J. Hilgen 1986 Event stratigraphy, basin evolution and tectonics of the Hellenic and Calabro-Sicilian arc. In: F.C. Wezel (ed.): The origin of arcs – Elsevier-Amsterdam: 327–350
- Nascimento, U. 1981 Lubricant and antilubricant effects of water – Lab. Nac. Eng. Civ. Mem. 560: 3–11
- Pieri, M. & L. Mattavelli 1986 Geologic framework of Italian petroleum resources – Am. Assoc. Pet. Geol. Bull. 70 (2): 103–130
- Pini, G.A. 1987 Studio geologica e analisi strutturale delle 'Argille Scagliose' del pedeappennino Bolognese – Ph. D. thesis Univ. Bologna and Modena: 207 pp
- Platt, J.P. 1986 Dynamics of orogenic wedges and the uplift of high-pressure metamorphic rocks – Geol. Soc. Am. Bull. 97: 1037–1053
- Raj, P.P. 1981 Comparison of true and residual friction angles – Soils Found. 21 (3): 99–103
- Ricci Lucchi, F. & S. D'Onofrio 1967 I rasporti gravitativi sedimentari nel Tortonianiano dell'Appennino Romagnolo – Giorn. Geol. 34: 1–30
- Ruggieri, G. 1970 Note illustrative della carta geologica d'Italia alla scala 1:100.000. Foglio 108 Mercato Saraceno – Serv. Geol. d'Italia: 56 pp
- Shimamoto, T. & J.L. Logan 1981 Effects of simulated clay gouges on the sliding behavior of Tennessee sandstone – Tectonophysics 75: 243–255
- Summers, R. & J. Byerlee 1977 A note on the effect of fault gouge composition on the stability of frictional sliding – Int. J. Rock Mech. Min. Sci. Geomech. Abstr. 14: 155–160
- Ten Haaf, E. 1985 A structural review of the Bolognese Apennines (with two field trip itineraries) – Giorn. Geol. 47 (1–2): 33–45
- Ten Haaf, E. & W.A. Van Wamel 1979 Nappes of the Alta Romagna. In: Van der Linden, W.J.M. (ed.): Fixism, mobilism or relativism: Van Bemmelen's search for harmony – Geol. Mijnbouw 58: 145–152
- Terzaghi, K. 1943 Theoretical soil mechanics – John Wiley (New York): 510 pp
- Treves, B. 1984 Orogenic belts as accretionary prisms: the example of the Northern Apennines – Ofioliti 9 (3): 577–618
- Treves, B. 1985 Mud volcanoes and shale diapirs. Their implications in accretionary processes. A review – Acta Nat. At. Parmense 21: 31–37
- Van den Berg, L. 1987 Experimental redeformation of naturally deformed scaly clays – Geol. Mijnbouw 65: 309–315
- Van Wamel, W.A., A.J. Bons, R.C.M.W. Franssen, W. Van Lingen, W. Postuma & A.C.A. Van Zutphen 1985 A structural geologic traverse through the Northern Apennines from Rapallo to Bettola (N. Italy) – Geol. Mijnbouw 64: 181–197
- Van Wamel, W.A. & P.E. Zwart 1990 On the structural geology and basin development of the Romagnan-Umbrian zone (upper Savio- and upper Bidente valleys, N. Italy) – Geol. Mijnbouw 69: 53–68
- Zanzucchi, G. 1980 I lineamenti geologici dell'Appennino Parmense – Vol. Mem. Sergio Venzo: 201–233