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Time dependent groundwater flow under river embankments

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Abstract

The Dutch Meuse – Rhine Delta mostly consists of Holocene clay and peat layers, overlying a Pleistocene sand substratum. The lowlands are below MSL, are protected against inundation by riverdikes and are mechanically drained. Time dependent variations of the river levels provoke time and distance dependent piezometric level responses in the Pleistocene sand layers. During storm surges, high piezometric pressures may reduce the bearing capacity of the embankment's foundation.

Predictions of the maximum piezometric pressures are needed for safe and economic design of the dikes, taking into account the unsteadiness of groundwater flow during design storm surge.

An analytical method allows to describe the piezometric response for both a tidal and step (surge) input. This method has been applied for the geo-hydrological conditions of the Meuse-Rhine delta: a pervious aquifer (Pleistocene stratum) overlain by an aquitard (Holocene layers) with time dependent leakage. The model accounts for the possible presence of a silt or mud layer between the river and the aquifer. If the calculated pressure in the sand somewhere exceeds the weight of the Holocene layers, pressure redistribution occurs and an area will be uplifted. The model can be adapted to this non-linearity and then allow to evaluate the time dependent length of the uplifted area and new piezometric levels.

The model parameters can be obtained from measurements made during normal tides.

Introduction

The Lower Meuse-Rhine delta consists mostly of Holocene layers of clay and peat, with a total thickness of approximately 8 to 12 metres, overlying a Pleistocene sand stratum with a thickness of approximately 20 to 40 metres (Fig. 1). The mean soil surface is several metres below mean sea level (MSL). The land is protected against inundation by artificial levees. The groundwater level is maintained below the soil surface by artificial drainage. Time dependent variations of the water level in the sea or river provoke time and distance dependent changes of the pore water pressure in the Pleistocene sand strata.

From time to time, the crests of the levees have to be raised and their slopes have to be fortified. The evaluation of their safety against embankment failure, caused by sliding of the inner slopes as a result of piping in or uplift of the Holocene layers, needs a reliable prediction of the pore pressure in the Pleistocene sand stratum under design storm conditions.

The classical prediction of the pore water pressure in the sand strata during storm surges, based on an assumed stationary ground water flow, yields maximal values. In most of the geo-hydrological situations existing in the Lower Meuse-Rhine delta, however, the period of a storm surge is too short to cause a stationary flow. Taking into account a

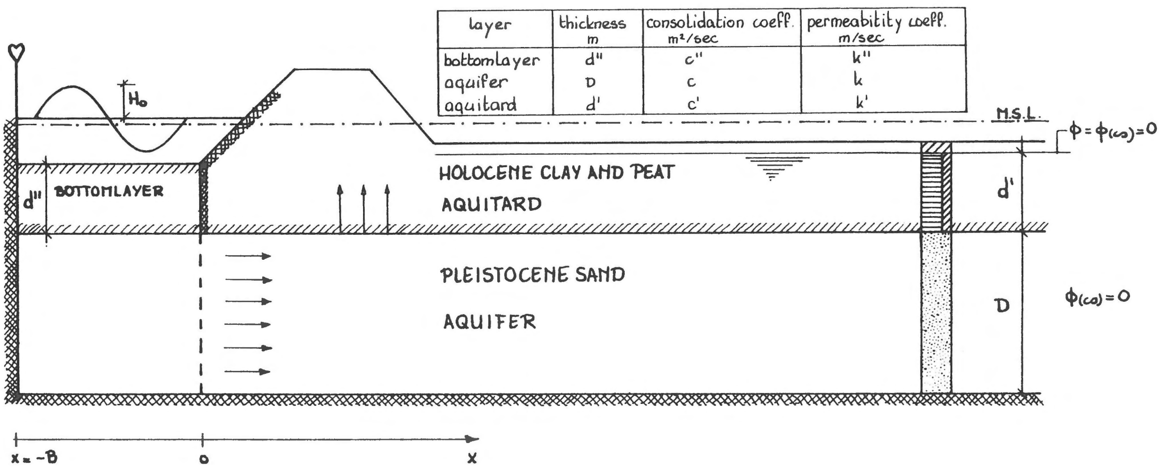

Fig. 1. General geo-hydrological situation in the lower Meuse-Rhine delta.

stationary flow during a storm surge, obviously leads to lower maximum pore water pressure in the Pleistocene aquifer system and consequently to a more economic design.

Therefore an analytical method was developed, capable of describing the piezometric response in the aquifer system, due to both a steady state (tidal) and step (surge) variation of the water level in the river or the sea. It has been applied for the most common geo-hydrological situation of the lower Meuse-Rhine delta, which is shown in Fig. 1: a pervious aquifer (Pleistocene sand) overlain by an aquitard (Holocene layers). The method accounts for the possible presence of a silt or mud layer between the river bottom and the aquifer (bottomlayer).

A geo-hydrological model for the time dependent flow

In general, an aquifer-aquitard model described as above is referred to as a leaky aquifer system. The contribution of the aquitard usually is defined by means of a constant leakage factor (Jacob, 1940). For time dependent flow however, it has been shown that this description is not correct (Barends, 1982). A more accurate description is based on the concept of a time dependent leakage factor. This concept is extended to describe the influence of compressible, low permeable silt or mud layers between the river bottom and the aquifer or the influence of shallow rivers, which do not directly contact the aquifer.

The semi-coupled analytical model presented here is an extension of Barends' model (1982) and links the time dependent storage and flow processes in the compressible bottom layer, the aquifer and the aquitard time dependently. The consolidation processes in the compressible layers and the flow and storage of water in the sand stratum are described by a set of four differential equations for time and place, which are coupled together to form the mathematical model. The coupling between the seepage flow in the aquifer and the consolidation in the bottom layer and in the aquitard is settled by the conditions at their common boundaries (interfaces): continuity of specific discharge and water pressure. The model is completed with the boundary conditions and the demand of compatible initial state conditions. This leads to a rather complicated set of equations. However, they can be solved with the aid of some simplifying assumptions:

- linear flow and linear elastic deformations;
- vertical flow only in the bottom layer and in the aquitard;
- horizontal flow only in the aquifer;
- the bottomlayer, the aquifer and the aquitard are homogenous;

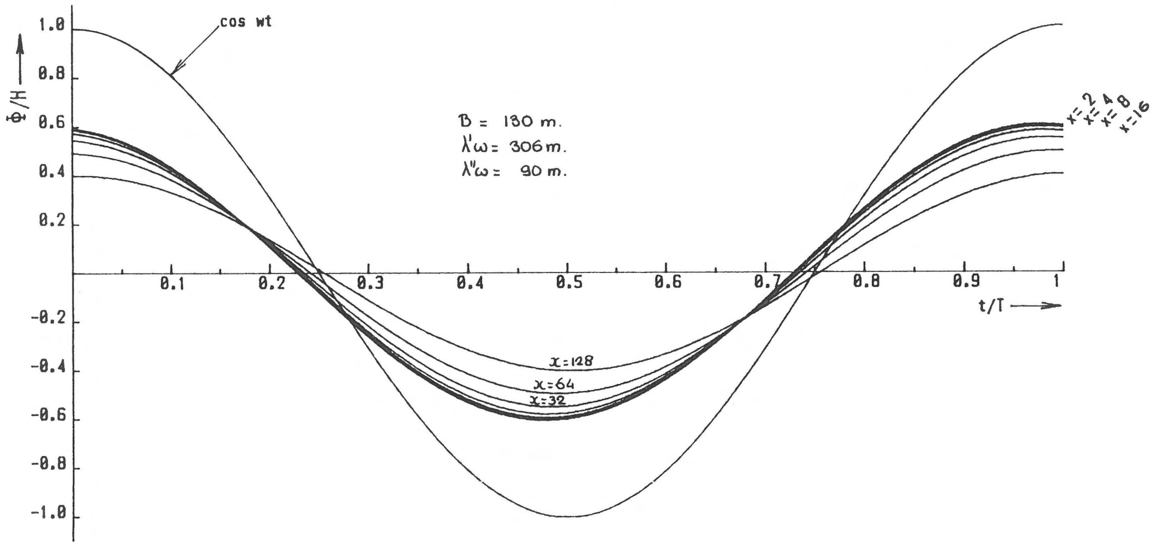

Fig. 2. Water pressure response on tidal variation of the water level at different x-values.

– the level of the phreatic line in the aquitard does not change.

The parameters of the different layers are summarized in Fig. 1. The thickness d' of the aquitard is measured between its boundary with the aquifer and the phreatic level. The riverbottom has a width equal to 2B. In the following, the reference potential head will be taken equal to zero on the phreatic level.

Response of the pore water pressure in the aquifer, due to tidal and surge variations of the water level

The equations describing the model can be solved for a sinusoidal input (tidal wave), using harmonic analysis and for a step input (sudden storm surge), using Laplace transform. The solutions are presented in Table 1, which also gives the solution for the stationary flow for comparison.

For the geo-hydrological conditions prevailing in the Lower Meuse-Rhine delta, the value of ω/c is very low compared to the other ones (Barends, 1982). Assuming ω/c tending to zero (incompressible sand layer), results in the formulas on Table 2. For B reaching an infinite value, the response due to a sinusoidal or step variation of the water level in a sea or a lake can be found.

Analysis of the response on a sinusoidal variation of the water level

As can be seen from equations 2 and 5, the response in the aquifer shows damping and phase shift, compared to the tidal variation of the water level. Both damping and phase shift are due to the consolidation process in the bottom layer, storage in the aquifer and consolidation in the aquitard.

The contribution of the bottom layer to the damping and phase shift, is independent of the distance to the river or the sea, but only depends on consolidation parameters and width and thickness values. The effect of the bottom layer on the phase disappears when the river becomes very wide (sea or lake), but its damping effect remains. It is interesting to observe that for a distance between the river at x = O to x = β (for the value of β see eq. 2 or eq. 5) the pore pressure in the aquifer preceeds the variations of the water level in the case of a river and reacts without time lag in the case of a sea or lake. The response first mentioned has been measured in the Lower Meuse-Rhine delta by De Lange et al. (1986).

The response of an aquifer-aquitard system without bottomlayer can easily be found from the mentioned formulas for f, g and β or λ''_ω equal to 0.

Table 1. Potential head in the aquifer (stationary flow, tide and sudden surge). All potential heads are referred to the phreatic level, which has a potential head equal to zero).

variation of water level	response in aquifer as function of distance and time
stationary flow (eq. 1)	$\varphi(x) = \dfrac{He^{-\frac{x}{\lambda'}}}{1 + \frac{\lambda''}{\lambda'} \coth(\frac{B}{\lambda''})}$ $\quad x>0$
	$\lambda' = (\frac{kDd'}{k'})^{0,5} \qquad \lambda'' = (\frac{kDd''}{k''})^{0,5}$
tide (eq. 2)	$\varphi(x,t) = \dfrac{H \cos(\omega t - x \sqrt{r'} \sin(\psi'/2 - \beta) \exp(- x \sqrt{r'} \cos(\psi'/2))}{[(1+f)^2 + g^2]^{0,5}}$ $\quad x>0$
	$r' = [(\frac{k'}{kD} \sqrt{(\frac{\omega}{2c'})})^2 + (\frac{\omega}{c} + \frac{k'}{kD} \sqrt{(\frac{\omega}{2c'})})^2]^{0,5}$
	$r'' = [(\frac{k''}{kD} \sqrt{(\frac{\omega}{2c''})})^2 + (\frac{\omega}{c} + \frac{k''}{kD} \sqrt{(\frac{\omega}{2c''})})^2]^{0,5}$
	$\psi' = \arctan\left(\dfrac{\frac{w}{c} + \frac{k'}{kD} \sqrt{(\frac{\omega}{2c'})}}{\frac{k'}{kD} \sqrt{(\frac{\omega}{2c'})}}\right)$
	$\psi'' = \arctan\left(\dfrac{\frac{w}{c} + \frac{k''}{kD} \sqrt{(\frac{\omega}{2c''})}}{\frac{k''}{kD} \sqrt{(\frac{\omega}{2c''})}}\right)$
	$\beta = \arctan(\frac{g}{1+f})$
	$f = (\frac{r'}{r''})^{0,5} \dfrac{\cos(\frac{\psi' - \psi''}{2}) \sinh \varrho + \sin(\frac{\psi' - \psi''}{2}) \cos \tau}{\cosh \varrho - \cos \tau}$
	$g = (\frac{r'}{r''})^{0,5} \dfrac{\sin(\frac{\psi' - \psi''}{2}) \sinh \varrho - \cos(\frac{\psi' - \psi''}{2}) \sin \tau}{\cosh \varrho - \cos \tau}$
	$\varrho = 2B \sqrt{r''} \cos(\psi''/2) \qquad \tau = 2B \sqrt{r''} \sin(\psi''/2)$
sudden surge (eq. 3)	$\varphi(x,t) = \dfrac{H \exp(- x/\lambda'_t)}{1 + \frac{\lambda''_t}{\lambda'_t} \coth(\frac{B}{\lambda''_t})}$ $\quad x>0$
	$\lambda'_t = [\frac{1}{2ct} + \frac{k'}{kD} \sqrt{(\frac{1}{2c't})} \coth(\frac{d'}{\sqrt{2c't}})]^{-0,5} \qquad \lambda''_t = [\frac{1}{2ct} + \frac{k''}{kD} \sqrt{(\frac{1}{2c''t})} \coth(\frac{d''}{\sqrt{2c''t}})]^{-0,5}$

Special case: no bottomlayer: $\lambda'' = 0$

$f = g = \beta = 0$

$\lambda''_t = 0$

Analysis of the response, due to a step variation of the water level

The response due to a step input (see equations 3 and 6), is illustrated in Fig. 3. One can see that the response becomes less with an increasing distance between the considered location and the river (or the sea). The gradient of the time-response curve is also reduced. The response is weaker when a bottomlayer is present because this lowers the discharge to the aquifer. In the case of an incompressible sand skeleton (c tending to an infinite value) the time dependent leakage factors become:

$$\lambda'_t = [\frac{k'}{kD} \sqrt{\frac{1}{2c't}} \cdot \coth(\frac{d'}{2c't})]^{-0,5}$$

$$\lambda''_t = [\frac{k''}{kD} \sqrt{\frac{1}{2c''t}} \cdot \coth(\frac{d''}{2c''t})]^{-0,5} \qquad \text{(eq. 7)}$$

When moreover the time t is very short compared to the hydrodynamic periods of the aquitard and

Table 2. Potential head in the aquifer (no elastic storage in the Pleistocene sand; all potential heads are referred to the phreatic level, which has $\varphi_{ref} - 0$).

variation of water level	response in aquifer as function of time and distance (no storage in sand)
stationary flow (eq. 4)	$\varphi(x) = \frac{H \exp(-x/\lambda')}{1 + \frac{\lambda''}{\lambda'} \coth(\frac{B}{\lambda''})}$
tide (eq. 5)	$\varphi(x,t) = \frac{H \cos(\omega t - x \tan(\pi/8)/\lambda_\omega - \beta) \exp(-x/\lambda'_\omega)}{[(1+m)^2 + n^2]^{0,5}}$
	$m = \frac{\lambda''_\omega}{\lambda'_\omega} \frac{\sinh(2B/\lambda''_\omega)}{\cosh(2B/\lambda''_\omega - \cos(2B\tan(\frac{\pi}{8})/\lambda''_\omega)})$
	$n = \frac{\lambda''_\omega}{\lambda'_\omega} \frac{\sin(2B\tan(\pi/8)/\lambda''_\omega}{\cosh(2B/\lambda''_\omega - \cos(2B\tan(\frac{\pi}{8})/\lambda''_\omega)})$
	$\beta = \arctan(\frac{n}{1+m})$
	$\lambda'_\omega = \sqrt{(\frac{kD}{k'})} \sqrt[4]{(\frac{c'}{\omega})} \frac{1}{\cos(\pi/8)}$
	$\lambda''_\omega = \sqrt{(\frac{kD}{k''})} \sqrt[4]{(\frac{c''}{\omega})} \frac{1}{\cos(\pi/8)}$
sudden surge (eq. 6)	$\varphi(x,t) = \frac{H \exp(-x/\lambda'_t)}{1 + \frac{\lambda''_t}{\lambda'_t} \coth(\frac{B}{\lambda''_t})}$
	$\lambda'_t = [\frac{k'}{kD} \sqrt{(\frac{1}{2c't})} \coth(\frac{d'}{\sqrt{(2c't)}})]^{-0,5}$
	$\lambda''_t = [\frac{k''}{kD} \sqrt{(\frac{1}{2c''t})} \coth(\frac{d''}{\sqrt{(2c''t)}})]^{-0,5}$
Special case: no bottomlayer: $\lambda'' = 0$, $\lambda''_\omega = 0$, $\lambda''_t = 0$	

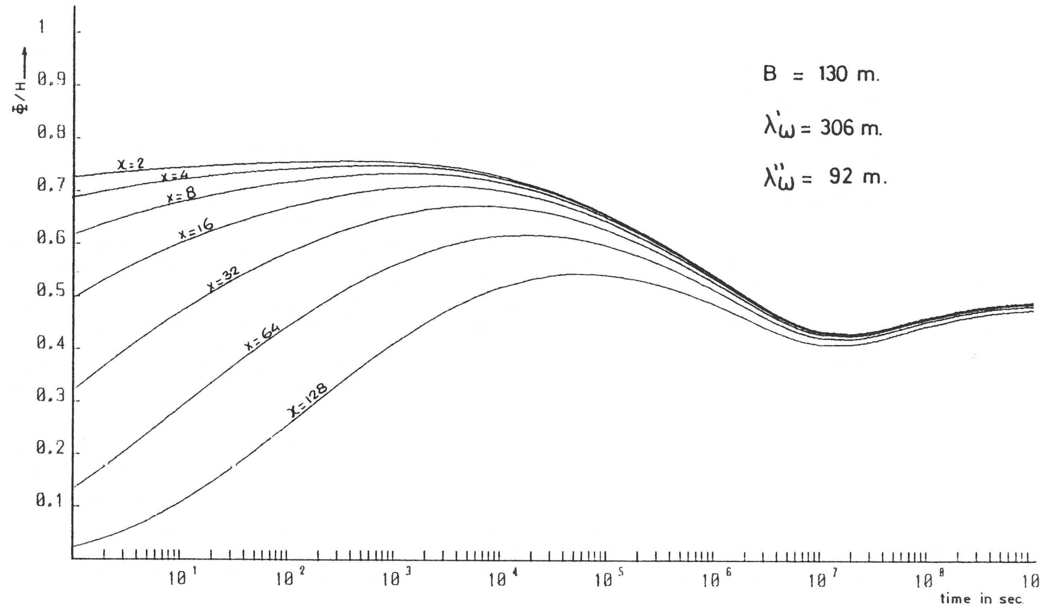

Fig. 3. Water pressure response on a step variation of the water level at different x-values.

the bottomlayer (equal to $\frac{2d'^2}{c'}$ and $\frac{2d''^2}{c''}$), the leakage factors for step variations of the water levels can be well approximated by:

$$\lambda_t' \approx \lambda_\omega' \sqrt[4]{4\omega t} \cdot \frac{1}{1{,}287}$$

$$\lambda_t'' \approx \lambda_\omega'' \sqrt[4]{4\omega t} \cdot \frac{1}{1{,}287} \qquad \text{(eq. 8)}$$

in which λ_ω' and λ_ω'' are the leakage factors for tidal flow.

Response due to an arbitrary variation of the water level

Closed analytical solutions with an acceptable accuracy for the response of the water pressure, due to variations of the water level other than the ones presented above are difficult to derive. However, since the soil parameters (permeability coefficients, consolidation coefficients) are assumed to be constant and the conditions at the boundaries between the layers and the boundary conditions do not vary, the superposition principle is applicable. Using this, one can evaluate the response due to an arbitrary variation of the water level by subdividing it into a series of positive and negative step functions shifted in the time. The superposition of the results of each step function forms the solution. The procedure is overlined in Fig. 4.

Prediction of the pore water pressure in the pleistocene sand substratum during design storm conditions

The objectives of the geo-hydrological model presented, are to provide a reliable way of predicting the water pressure in the soil layers during a design storm and to provide a method to estimate the necessary soil parameters, using measurements of the water pressure response under normal variations of the water level in the river or sea. In the Lower Meuse-Rhine Delta, the maximum water level for design conditions can be subdivided into three components: A stationary one, a tidal one (due to storm tide in the North Sea) and one with a more or less block shape (due to high river discharge). The duration of this combined maximal water level is generally short compared with the

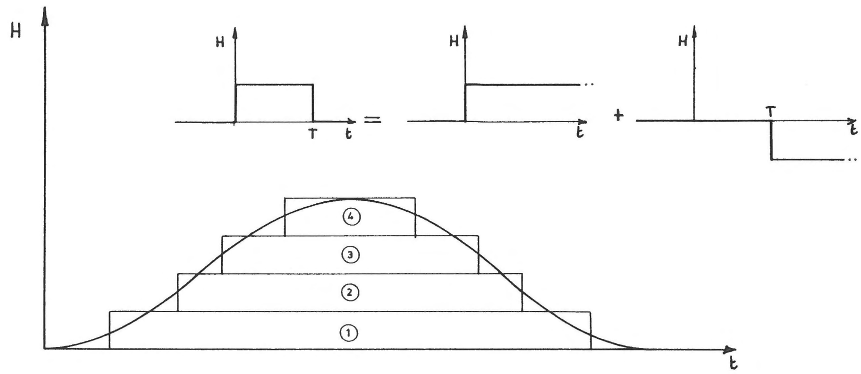

Fig. 4. Superposition principle to evaluate the response, due to an arbritary variation of the water level.

hydrodynamic periods of the bottom layer and the aquitard.

During normal conditions, the water level is composed of a stationary (mean water level) and a tidal component. Based on this, a useful procedure can be outlined (see Fig. 5). Piezometers are installed into the aquifer in at least two, but preferably three locations with sufficient distance from one to the other.

The analysis of the measured response and the prediction of the response under design flood can be performed using the formulas given in Table 2 as the consolidation of the aquitard and the bottom layer are dominating the time-lag observed in the transient response of leaky aquifers (Barends, 1982). Using the damping of the response of the piezometers one to the other (e.g. response of piezometer 2 on piezometer 1 etc.) and equation 5, one can find the leakage factor of the aquitard for tidal variations of the water level λ'_ω:

$$\lambda'_\omega = \frac{x_{ij}}{\ln \frac{\varphi_i}{\varphi_j}} \qquad \text{(eq. 9a) or}$$

$$\lambda'_\omega = \frac{\sum x_{ij}^2}{\sum x_{ij} \ln \frac{\varphi_i}{\varphi_j}} \qquad \text{(eq. 9b)}$$

in which: x_{ij} is the distance between piezometers i and j and
φ_i, φ_j is the measured piezometric level

Equation 9b follows from the least squares method.

Based on λ'_ω and the response of the piezometers on the tidal variation of the water level, one can find λ''_ω by solving equation 5 (or the equivalent one in the case of a sea). The using λ'_ω and λ''_ω, one can calculate λ'_t and λ''_t at every moment, using equation 8, provided that the duration of the surge is short, compared to the hydrodynamic period. The leakage factors λ' and λ'' for stationary flow, follow in the same way from the measured mean water and piezometric levels. Care must be taken with large scale seasonal effects which can affect the mean piezometric level. Subsequently, with the geo-hydrological parameters found as described above, one can predict the water pressures in the aquifer for the design storm, i.e. for the expected ultimate water level and its duration, using the formulas corresponding to the chosen geo-hydrological model.

Uplift of the aquitard

A basic assumption underlying all above mentioned formulas, is that the water pressure in the aquifer can increase without being limited by the weight of the overlying aquitard. When the water pressure somewhere in the sand layer indeed tends to exceed the vertical pressure exerted on the boundary by the weight of the flood bank and the foundation layers no further development of the

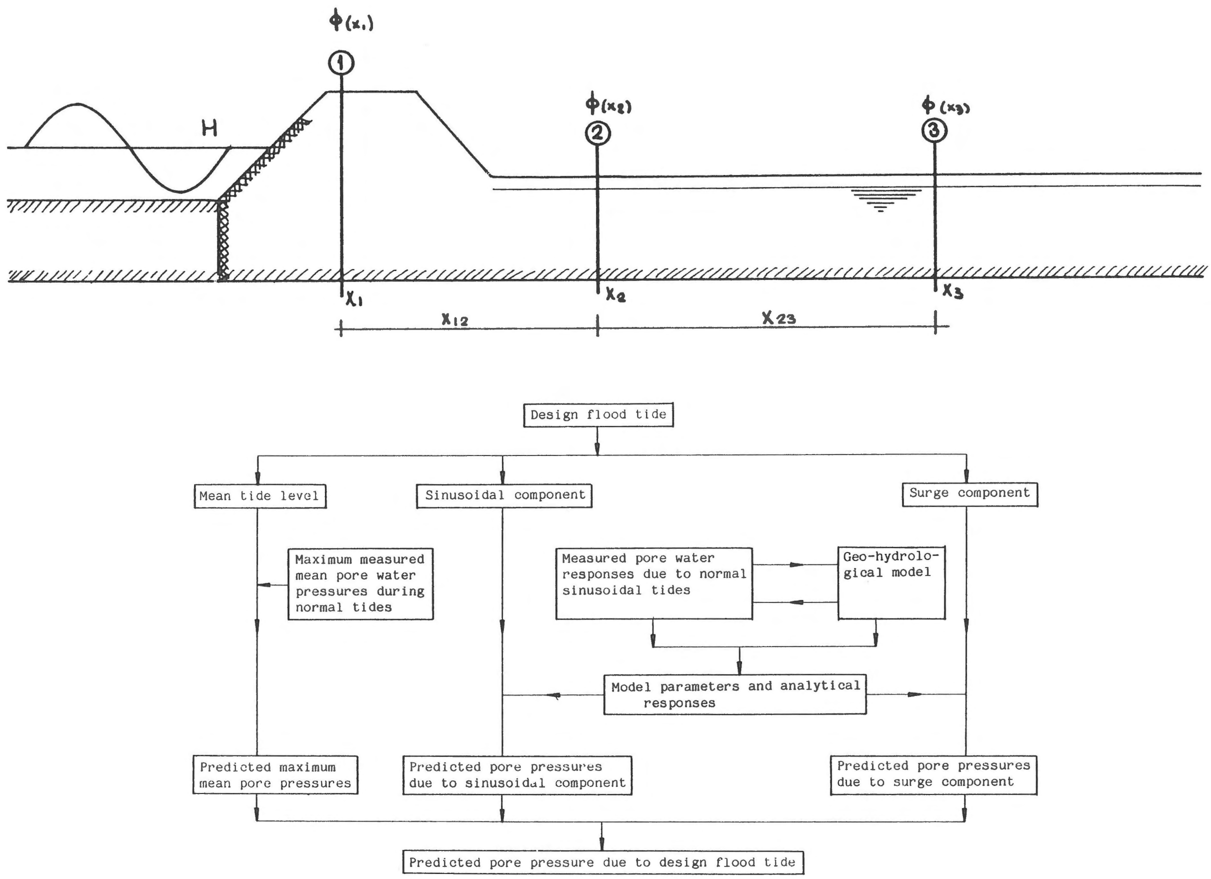

Fig. 5. Outline of the method for prediction of maximum pore pressure, likely to occur during design flood conditions. a. Location of the piezometers. b. Calculation flowchart (after Marsland & Randolph, 1978).

water pressure in this area is possible. In such a case, the formulas are no longer valid because a pressure redistribution will occur. As a result, the water pressure in the aquitard will nowhere exceed the local total vertical stress at the separation between the aquifer and the aquitard.

The area in which the water pressure equals the total vertical stress, the Holocene layers of clay and peat will be uplifted. The significance of uplift on the stability and deformations of levees has been outlined by Marsland (1961), Marsland & Randolph (1978), Padfield & Schofield (1983) and Teunissen et al. (1986): The development of a large uplift aera near the toe of a levee can lead to large deformations or even collapse of the embankment because uplift reduces the shear forces transmitted to the stiff sand stratum (see Fig. 6).

The influence of an uplifted area on the deformations and on the stability of a levee, depends on its location and magnitude, its impact on the existing force equilibrium and the stiffness of the soil near the toe of the levee. The equations mentioned in Tables 1 and 2 give the variations of the piezometric head in the aquitard in response to the variation of the piezometric waterhead of outside the flood embankment. These variations of piezometric heads are independent of the chosen referential piezometric head. As *pressures* have to be compared when considering uplift phenomena, a suitable choice of the referential piezometric head is needed in order to convert the piezometric heads into waterpressures which can directly be compared to the pressure exerted by the Holocene layers on the sand stratum. Taking the reference potential head

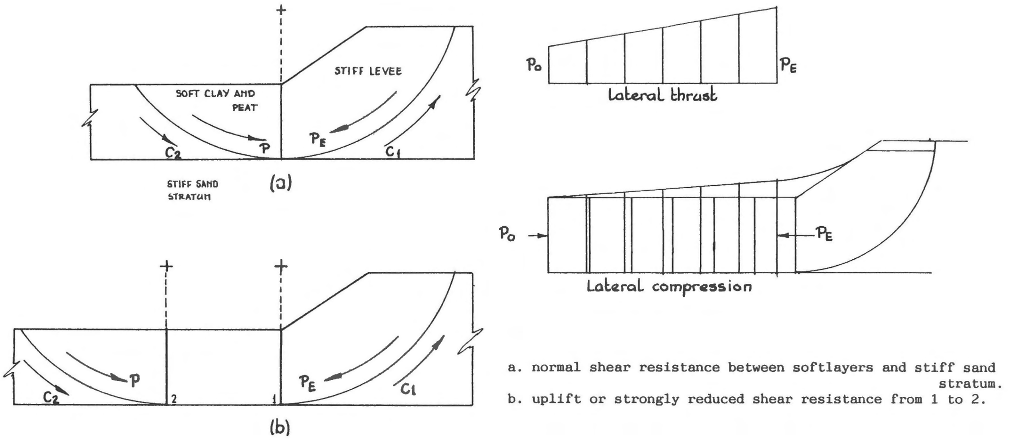

Fig. 6. Dike failure, caused by excessive horizontal deformation of the passive area under uplift conditions (after Padfield & Schofield, 1983).

equal to zero on the phreatic level in the polder, makes direct comparison between the soil pressures and the waterpressures possible: the stationary potential head given in the equation 1 and 4 with $\varphi_{ref} = O$, is also the expression of the stationary waterpressure at the bottom of the Holocene layers if the waterlevel H_1 is expressed in metres above the phreatic level and multiplied by the unit weight of the pore water, γ_w.

As equations 2, 3, 5 and 6 are variations of the potential head, they are also variations of the waterpressure when multiplied by γ_w. Hence, the waterpressure becomes the sum of the stationary pressure referred to above and the time dependent pressure variation. This choice of the reference potential can be made because the phreatic level is considered as being constant during the phenomena which are analyzed. In case of important phreatic storage in the Holocene layers, this would not be true. For typical Dutch dike cross sections in the lower Meuse-Rhine delta, the beginning of the uplifted area (at x = P, see Fig. 7a) can approximately be taken at the x-value where $\gamma_w . \varphi(x, t)$ equals the total vertical stress $\sigma_v(x)$ for the lowest t-value (see the line corresponding with t2 in Fig. 7a). φ (x,t) is calculated with the formulas on Table 1 or 2. The total vertical stress includes the weight of the clay and peat layers and the weight of the flood bank.

The moment the water pressure in the aquifer somewhere equals the vertical stress, the water pressure will no longer increase in that place, but (locally) will remain equal, see Fig. 7b. The uplift area will develop as a function of time, starting from the value of x = P according to the formula given in Table 3. This table also gives the waterpressure in the aquifer from the moment of the beginning of the uplift conditions. The water pressure distribution during the uplift conditions as a function of the distance is given in Table 3 and illustrated by the line corresponding with t3 in Fig. 7a. It is emphasized that the formulas given in Table 3 are approximations. Their accuracy has not been verified yet, because up till now no correct mathematical solution has been found, not analytical, nor numerical. Further research is needed to verify the accuracy of the proposed solutions.

It is interesting to observe that all the geo-hydrological parameters needed to evaluate the magnitude of the uplift area can be obtained from measurements made over normal tides as explained above. The pressure redistribution in uplift conditions however, shows that the determination of λ'_ω and λ''_ω is not possible if uplift conditions occurred during the measurement of the water pressure response due to the (normal) tidal variation of the water level. The superposition principle as explained earlier is no longer more valid when uplift occurs as uplift is a typical non-linearity in the conditions at the separations between the aquifer and the aquitard.

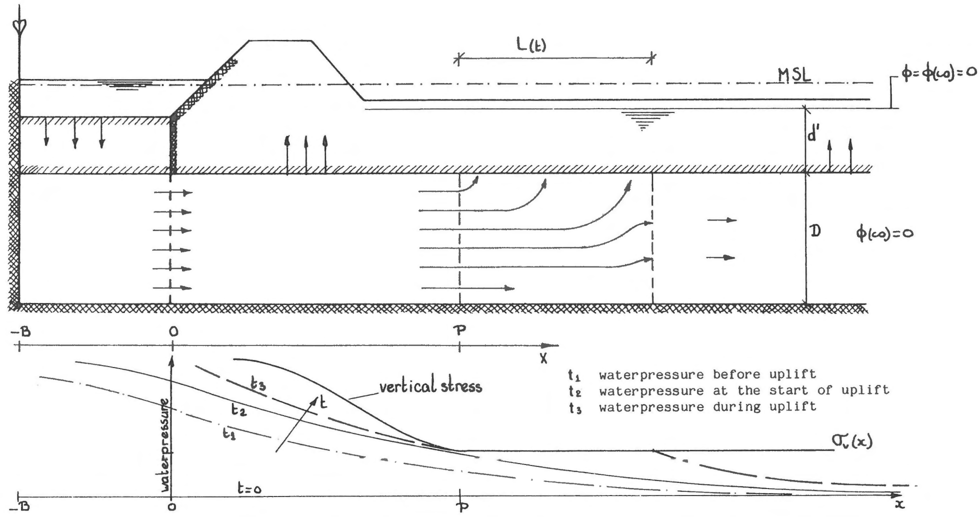

Fig. 7a. Water pressures distribution before and during uplift.

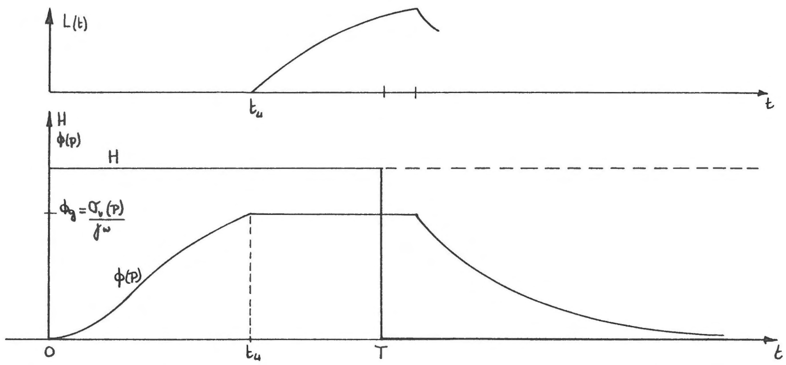

Fig. 7b. Water pressure development and length of the uplift area as a function of time.

Case study

The method outlined above is applied to the measurement of the tidal responses in three piezometers located at 16 m, 60 m and 115 m from the rivershore (Fig. 9).
The actual river width is about 260 m. Using the response of one piezometer to the other (eq. 9), one finds $\lambda'_{\omega} = 306$ m. Then, by trial and error, one finds the corresponding value of $\lambda''_{\omega} = 90$ m. The calculated and the measured values of the damping are shown in the Table in Fig. 8. They are fitting very well. These values where used to plot Figs. 2 and 3 (response due to tidal variation and a sudden surge) and Fig. 8 (length of the uplift area). Knowing the total vertical stress acting on the aquifer and the design water level, one can determine the water pressure in the aquitard and determine whether uplift is likely to occur during the design storm. One can calculate the extent of the uplift area using the equations given in Table 3.

piezometer no.	distance from river m	measured damping -	calculated damping -
1	16	0,727	0,729
2	60	0,636	0,634
3	115	0,500	0,501

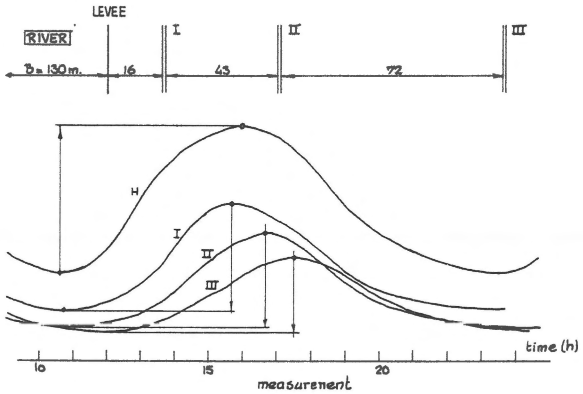

Fig. 8. Case study: Measured and calculated damping (λ'_ω = 306 m; λ''_ω = 90 m)

Conclusion

A method is presented here, to assess λ'_ω and λ''_ω, the geohydrological parameters of an aquifer-aquitard system with a bottomlayer between the river and the aquifer. This is done with the aid of the measurement of the response of the water pressure in the aquifer, due to normal tidal variations of the river level. The parameters can be directly used to predict the water pressure during design storm conditions. If the water pressure exceeds the pressure due to the weight of the layers of the aquitard, uplift will occur. The same geohydrological parameters can be used to calculate approximately the length of the uplifted zone and the new distribution of the water pressures.

Table 3. Approximate length of the uplifted aera and potential head in the aquifer during uplift conditions reference potential head is phreatic level, which has $\varphi_{ref} = 0$; incompressible aquifer).

$$L(t) = \frac{2D}{\pi} \text{Arc cosh} \left\{ \frac{1}{\sin \left[\frac{\pi}{2} \frac{\varphi_g \sinh (P/\lambda'_t)}{\varphi (x=0) - \varphi_g \cosh (P/\lambda'_t)} \right]} \right\} \quad \text{(eq. 10)}$$

$$\varphi(x=o,t) = \frac{H + \frac{\lambda''_t \coth (B/\lambda''_t)}{\lambda'_t \sinh (P/\lambda'_t)} \varphi_g}{1 + \frac{\lambda''_t}{\lambda'_t} \coth (B/\lambda''_t) \coth (P/\lambda'_t)}$$

$$\lambda'_t = \left[\frac{k'}{kD} \sqrt{\left(\frac{1}{2c't}\right)} \coth \left(\frac{d'}{\sqrt{2c't}}\right)\right]^{-0,5}$$

$$\lambda''_t = \left[\frac{k''}{kD} \sqrt{\left(\frac{1}{2c''t}\right)} \coth \left(\frac{d''}{\sqrt{2c''t}}\right)\right]^{-0,5}$$

$0<x<P$

$$\varphi(x,t) = \varphi(x=o,t) = \frac{\sinh\ (P-x)/\lambda'_t}{\sinh (P/\lambda'_t)} + \varphi_g \frac{\sinh (x/\lambda'_t)}{\sinh (P/\lambda'_t)}$$

$P<x<P+L$

$$\varphi(x,t) = \varphi_g$$

$P+L<x$

$$\varphi(x,t) = \varphi_g \exp (-(x-(P+L))/\lambda'_t)$$

Special case: no bottomlayer: $\lambda''_t = 0$

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