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ESTIMATION OF HEAT FLOW IN OIL WELLS BASED ON A RELATION BETWEEN HEAT CONDUCTIVITY AND SOUND VELOCITY[1]

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ABSTRACT

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Based on published data, it is assumed that the ratio of sound velocity to thermal conductivity exhibits a linear relationship with formation temperature for most sedimentary rocks. Combination of this assumption with Fourier's heat-flow law yields

$$q^* = \ln \frac{T_L + c}{T_u + c} \cdot \frac{1}{A(t_L - t_u)}$$

where T_L and T_u are the subsurface temperatures at the top and the bottom of an interval, respectively, t_L and t_u the sound travel times, and q^* is the heat flow. This relation has been tested in the case of 10 wells, for which accurate data were available. The relation generated very satisfactory fits with the measured data for siliciclastic and carbonate rocks. The parameters a and c take respective values of 1.039 and 80.031; heat flow (q^*) is expressed relative to the heat flow in the standard well Bolderij-1 in the Groningen gasfield (Bolderij Unit, BU).

A method for estimating the relative heat flow from bottom-hole temperatures as observed during logging operations, and sound-travel times from well-shoot in combination with sonic-log data, has been developed and tested in the Viking and Central grabens of the UK sector of the North Sea. In this region the mean relative heat flow using data from 120 wells is 0.601 BU, with a standard deviation of 0.055 BU.

Comparisons of calculated relative heat-flow values in BU, with heat-flow values in SI-Units conventionally obtained suggests that the Bolderij Unit is equivalent to about 77 mWm^{-2}.

INTRODUCTION

For geothermic investigations in sedimentary basins it would be desirable to make use of the large amount of data acquired by drilling for oil. There are, however, two main difficulties in using these data. Firstly, common to all subsurface investigations by drilling, the downhole thermal conductivities are not accurately known. Secondly, most of the temperature data are socalled 'bottomhole measurements' recorded during short intervals in drilling operations. These time intervals are too short for the disturbed subsurface temperatures to completely recover their predrilling values.

The measured conductivities of sedimentary rocks have wide ranges, even for restricted lithotypes, and the corrections required to obtain reliable down-hole values from laboratory measurements are not properly known. We present the results of an attempt to overcome these difficulties by assuming an empirical relationship between thermal conductivity, acoustic velocity and temperature. On this basis an equation for heat flow, in terms of subsurface temperature and one-way sound travel-time from the surface, has been derived. This equation has been successfully tested using accurate subsurface-temperature data and sound travel-time data from a set of wells.

The heat flow of these wells can be accurately calculated relative to the heat flow in a standard well.

A description is given of a method for estimating relative heat-flow values using the more commonly available bottomhole temperatures observed during well-logging operations. A geothermic investigation of part of the North Sea basin is presented as an example.

Finally, we present an attempt to calibrate the relative heat-flow model in SI units.

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THE MODEL

KARL (1965) analysed thermal-conductivity (K) and acoustic-velocity (V) data and established the relationship

$$\frac{V}{K} = \text{constant} \tag{1}$$

for most sedimentary rocks with water-filled pores at room temperature.

Thermal conductivities are known to depend strongly on temperature (cf. KARL, 1965; SCHATZ & SIMMONS, 1972; CLARK, 1966), as do sound velocities to a much lesser extent (KARL, 1965). Experimental measurements of thermal conductivity at different temperatures (SCHATZ & SIMMONS, 1972; CLARK, 1966) suggest that the relation between thermal conductivity (K) and temperature (T) is of the form

$$K = \frac{1}{a'T + b'} \tag{2}$$

a′ and b′ being constants.

Equation (2) in combination with equation (1) implies that

$$\frac{V}{K} = aT + b, \tag{2a}$$

a and b also being constants.

To test this last relation it should, in principle, be possible to measure sound velocity and thermal conductivity at different temperatures on a large number of core samples. However, such a test would be very labour-intensive, costly and fraught with experimental difficulties.

An alternative method of testing the applicability of the relation has been adopted based on subsurface-temperature and sound travel-time data from oil wells.

For convenience, the equation (2a) can be re-arranged, giving

$$\frac{V}{Ka} = c + T, \tag{3}$$

where c = b/a.

The vertical flux of heat through a rock column is given by Fourier's law

$$q = -K\frac{\delta T}{\delta x}\,. \tag{4}$$

The minus sign in equation (4) implies heat flow in the direction of increasing depth, x, measured downward from the surface. For convenience, we remove the negative and we take positive q values to mean heat flow towards the surface of the earth from depth.

Combining (3) and (4) gives

$$aq = \frac{V}{c + T}\cdot\frac{\delta T}{\delta x}\,. \tag{5}$$

Introducing seismic travel time, $\frac{\delta T}{\delta x}$ can be transformed by the relation

$$\frac{\delta T}{\delta x} = \frac{\delta T}{\delta t}\cdot\frac{\delta t}{\delta x} = \frac{\delta T}{\delta t}\cdot\frac{1}{V},$$

where t is the one-way travel time from the surface.

Substituting this relation into (5) yields

$$aq = \frac{1}{c + T}\cdot\frac{\delta T}{\delta t}, \tag{6}$$

where a and c are constants, and for steady-state solutions q is a constant. Separating variables and integrating between limits t_u and t_L and T_u and T_L

Table I
Equilibrium temperature and travel-time data for the calibration wells.

Well	Depth, m	One-way travel time, s	Temperature, $^{\circ}$C
Bolderij-1 Groningen gas field (Netherlands)	600	0.31665	26.6
	900	0.4506	43.0
	1300	0.5759	59.55
	1500	0.6273	67.2
	1800	0.7011	79.3
Ten Post-1 Groningen gas field (Netherlands)	600	0.3204	27.0
	800	0.4242	40.5
	1050	0.5117	51.5
	1300	0.5891	62.6
	1650	0.6742	76.8
Haarle-1 Twente (Netherlands)	594	0.315	26.6
	794	0.3825	33.9
	994	0.4375	40.7
	1194	0.490	48.0
Nun River-1 (Nigeria)	2057.4	0.884	43.33
	2133.6	0.918	45.0
	2286.0	0.973	48.89
	2438.4	1.028	53.33
	2590.8	1.082	56.67
	2743.2	1.132	60.0
	2895.6	1.181	64.44
Ogini-1 (Nigeria)	1504.8	0.688	44.44
	1809.6	0.800	53.33
	2266.8	0.955	66.11
	2724.0	1.095	78.89
Imo River-1 (Nigeria)	731.5	0.386	31.11
	896.1	0.458	35.0
	1200.9	0.592	45.0
	1505.7	0.700	52.78
	1810.5	0.870	64.44
	2115.3	0.925	75.0
	2724.9	1.117	86.11
Ebubu-5 (Nigeria)	914.4	0.449	35.56
	1219.2	0.589	42.22
	1524.0	0.711	50.0
	1828.8	0.831	56.11
	2133.6	0.942	62.78
	2438.4	1.050	70.0
	2718.8	1.147	77.78
Al Huwaisah-1 (Oman)	642.5	0.233	50.56
	971.7	0.354	63.33
	1216.8	0.430	70.56
	1322.2	0.516	81.11
Saih Rawl-2 (Oman)	422.2	0.137	42.78
	720.9	0.257	51.67
	894.6	0.348	58.33
	1373.0	0.491	72.22
	2050.0	0.641	87.22
	2702.0	0.754	98.33
Shell Chapman-1 (Texas, USA)	914.4	0.408	51.5
	1219.2	0.534	63.6
	1524.0	0.653	75.0
	1828.0	0.762	87.2
	2133.6	0.864	99.6
	2438.4	0.949	109.1
	2743.2	1.035	117.9
	3048.0	1.111	130.3
	3352.8	1.194	141.0
	3657.6	1.269	149.2

Table IIa
Cross-correlation of relative heat flows (BU) with standard deviations (calibration data in table IIb).

	Country	Netherlands			Nigeria				Oman		U.S.A.	Mean of std.devs. temperature residuals for all wells in calibration row	Mean of std.devs. heat flow for all wells in calibration row	Row number
Well	Well	Bolderij-1	Ten Post-1	Haarle-1	Nun River-1	Ogini-1	Imo River-1	Ebubu-5	Al Huwaisah-1	Saih Rawl-2	Shell Chapman-1			
Bolderij-1		1.000 0.007	1.029 0.013	1.008 0.022	0.512 0.010	0.576 0.003	0.543 0.024	0.427 0.007	0.709 0.017	0.590 0.008	0.621 0.008	0.683	0.012	1
Ten-Post-1		- 0.007	1.029 0.013	1.005 0.022	0.513 0.010	0.577 0.003	0.543 0.024	0.427 0.007	0.710 0.017	0.592 0.007	0.625 0.008	0.680	0.012	2
Haarle-1		- 0.032	1.031 0.034	1.132 0.000	0.506 0.010	0.543 0.012	0.521 0.030	0.416 0.007	0.654 0.019	0.535 0.011	0.503 0.013	1.080	0.017	3
Nun River-1		- 0.013	1.029 0.018	1.049 0.015	0.510 0.010	0.563 0.006	0.534 0.026	0.423 0.006	0.688 0.017	0.569 0.008	0.572 0.010	0.784	0.013	4
Ogini-1		- 0.008	1.028 0.013	0.992 0.024	0.514 0.010	0.582 0.002	0.546 0.024	0.429 0.008	0.718 0.017	0.600 0.008	0.644 0.007	0.681	0.012	5
Imo River-1		- 0.020	1.027 0.022	0.943 0.031	0.518 0.011	0.601 0.007	0.558 0.023	0.435 0.011	0.751 0.020	0.634 0.013	0.732 0.008	0.847	0.017	6
Ebubu-5		- 0.019	1.030 0.022	1.073 0.011	0.509 0.010	0.557 0.008	0.530 0.027	0.421 0.006	0.677 0.018	0.558 0.008	0.549 0.012	0.866	0.015	7
Al Huwaisah-1		- 0.007	1.029 0.013	1.005 0.022	0.513 0.010	0.577 0.003	0.543 0.024	0.427 0.007	0.710 0.017	0.592 0.008	0.625 0.008	0.629	0.012	8
Saih Rawl-1		- 0.008	1.029 0.014	1.021 0.020	0.511 0.010	0.572 0.004	0.540 0.025	0.426 0.007	0.702 0.017	0.583 0.007	0.598 0.009	0.652	0.012	9
Shell Chapman-1		- 0.015	1.028 0.018	0.963 0.028	0.516 0.011	0.593 0.004	0.553 0.023	0.432 0.009	0.737 0.018	0.619 0.011	0.693 0.007	0.750	0.014	10
Overall means and std.dev. of means			1.029 0.001	1.024 0.049	0.512 0.003	0.574 0.017	0.541 0.011	0.426 0.005	0.705 0.029	0.587 0.029	0.616 0.066			11
Range of depths (m)		600-1800	600-1650	594-1194	914-2719	1505-2724	743-2725	914-2719	643-1522	422-2702	914-3658			12
Range of travel times (s)		0.317-0.701	0.314-0.674	0.315-0.490	0.884-1.181	0.688-1.095	0.386-1.117	0.449-1.05	0.233-0.516	0.137-0.754	0.217-1.269			13
Range of temperature (°C)		27-29	27-77	27-48	43-64	44-79	31-86	36-78	51-81	43-98	36-149			14
Main lithologies		Chk-Mrl	Chk-Mrl	Sh	Sh-Snd	Sh-Snd	Sh-Snd	Sh-Snd	Lstn-Dol-Sh	Lstn-Dol-Sh	Sh-Snd			15

$$aq \int_{t_u}^{t_L} dt = \int_{T_u}^{T_L} \frac{1}{c + T}\, dT$$

yields

$$q = \frac{\ln\left\{\dfrac{c + T_L}{c + T_u}\right\}}{a(t_L - t_u)} \tag{7}$$

which is the required relationship.

It can be re-arranged to give the temperature at the jth depth level, T_j, in terms of travel time to the jth level, t_j, and temperature and travel time for some fixed higher level (T_o, t_o), heat flux q, and coefficients a and c

$$T_j = T_o e^{aq(t_j - t_o)} + c(e^{aq(t_j - t_o)} - 1). \tag{8}$$

We have solved equations of this form by a non-linear least-squares technique, using (T_j, t_j) data from oil wells, treating travel time and temperature as the independent and dependent variables, respectively.

The data used are given in table I. The temperature data are from equilibrium-temperature surveys carried out after the well had been shut in for a period of more than six weeks. The one-way sound travel-times from the surface were established by well-shoot surveys, where necessary combined with sonic-log data. The main lithologies encountered in the used part of these wells are given in the bottom row of table II. Data from the evaporite series at the bottom parts of the wells Bolderij-1, Ten Post-1, Haarle-1, Al Huwaisah-1 and Saih-Rawl-1 have not been included.

Table IIb
Calibration data for rows of table IIa.

Well	Number of data points	$A = a.q^*$	Std. dev. A	C = b/a	Std. dev. C	Std.dev. temper. residuals	
Bolderij-1	5	1.0390184	0.1182946	80.030606	14.877608	0.271	1
Ten Post-1	5	1.0433057	0.2289878	83.219092	29.321777	0.473	2
Haarle-1	4	2.1884872	0.0005522	19.258517	0.014076	0.000	3
Nun River-1	7	0.6956953	0.4631683	48.760722	67.84495	0.367	4
Ogini-1	4	0.5255629	0.0703801	99.615495	21.516910	0.015	5
Imo River-1	7	0.2535838	0.3957077	248.07931	475.95125	2.112	6
Ebubu-5	7	0..6477601	0.1374000	37.4355508	19.629809	0.506	7
Al Huwaisah-1	4	0.7212061	0.5275505	83.064322	108.34367	0.519	
Sai Rawl-1	6	0.6685392	0.1456091	67.713322	29.74300	0.595	9
Shell Chapman-1	10	0.4438937	0.0772136	161.26191	45.14616	0.930	10

Our first method of solution is to set the coefficient A equal to aq, since for one well both a and q are constants, and to determine A, c and T_o values separately for each well. T_o is the unknown surface temperature, where $t_o = O$. We thus take into account transient thermal effects near the surface due to, for example, Quaternary permafrost conditions. These individually calibrated values of A and c, which vary widely for different wells (Table II), are then used to determine q and T_o

Table III
Cross-correlation of surface-temperature estimates (°C) with standard deviations.

	Country	Netherlands			Nigeria				Oman		U.S.A.	
	Well	Bolderij-1	Ten Post-1	Haarle-1	Nun River-1	Ogini-1	Imo River-1	Ebubu-5	Al Huwaisah-1	Saih Rawl-2	Shell Chapman-1	
Calibration wells												
Bolderij-1		- 3.206 0.319	- 3.830 0.561	- 3.547 0.723	- 3.141 0.829	2.502 0.228	9.453 1.877	14.380 0.623	30.069 0.790	32.475 0.471	21.981 0.877	1
Ten Post-1		- 3.501 0.324	- 4.134 0.566	- 3.751 0.748	- 3.468 0.841	2.192 0.217	9.254 1.882	14.233 0.637	29.976 0.793	32.393 0.477	21.660 0.861	2
Haarle-1		6.353 0.975	5.966 0.959	3.757 0.0003	7.052 0.541	11.898 0.676	15.897 1.793	19.226 0.458	33.019 0.826	35.128 0.659	30.819 1.345	3
Nun River-1		0.518 0.520	- 0.003 0.651	- 0.864 0.415	0.922 0.694	6.306 0.421	11.963 1.824	16.246 0.488	31.228 0.771	33.512 0.471	25.759 1.080	4
Ogini-1		- 4.844 0.403	- 5.517 0.618	- 4.669 0.866	- 4.968 0.898	0.764 0.196	8.348 1.912	13.566 0.706	29.553 0.815	32.017 0.516	20.154 0.796	5
Imo River-1		-10.707 1.220	-11.568 1.273	- 8.424 1.406	-11.715 1.208	-5.806 0.659	4.389 2.131	10.686 1.084	27.674 0.999	30.363 0.847	12.597 0.894	6
Ebubu-5		2.391 0.679	1.917 0.751	0.561 0.272	2.920 0.638	8.147 0.515	13.225 1.810	17.194 0.454	31.806 0.780	34.031 0.515	27.486 1.175	7
Al Huwaisah-1		- 3.487 0.324	- 4.119 0.565	- 3.742 0.747	- 3.452 0.840	2.207 0.218	9.263 1.882	14.240 0.637	29.981 0.793	32.397 0.477	21.675 0.862	8
Saih Rawl-2		- 1.929 0.344	- 2.526 0.563	- 2.655 0.615	- 1.744 0.779	3.819 0.289	10.307 1.855	15.012 0.568	30.466 0.778	32.829 0.455	23.322 0.946	9
Shell Chapman-1		- 8.193 0.818	- 8.972 0.927	- 6.861 1.170	- 8.781 1.064	-2.919 0.392	6.088 2.018	11.915 0.908	28.487 0.902	31.076 0.680	16.051 0.738	10
Overall means and std. devs. of these means		- 2.662 4.935	- 3.279 5.073	- 3.020 3.520	- 2.642 5.421	2.911 5.124	9.819 3.329	14.669 2.473	30.226 1.546	32.622 1.378	22.150 5.281	11

for each well in turn. For each calibration set of A, c values, we calculate relative heat-flow values q*, with reference to the particular calibration well, i.e. relative heat flow in the calibration well is unity. For comparison, in the cross-correlation study the values of q* have been scaled to a common reference unit -the Bolderij Unit or BU- i.e. relative heat flow in the well Bolderij-1 ($q^* = 1.0$ BU).

This statistical exercise is summarized in tables II and III for heat flows in BU and surface temperatures, respectively. The calibration-well data are arranged in rows, and the values calculated for each well using the different calibrations are reported in columns for ease of comparison.

As can be seen, the independent calibrations produce almost identical results for heat flow, despite the fact that the wells were drilled through several different lithologies (Table II). The estimated heat flows have very small standard deviations: less than 0.035 BU.

The estimates of surface temperature are more variable (Table III) but, bearing in mind the inaccuracy of the temperature-survey data, these deviations are not excessive. The values are negative for The Netherlands and for one of the Nigerian wells. We consider the low estimated T_0 values in The Netherlands to reflect recent permafrost conditions. The variable estimates for Nigeria probably result from the cooling influence, near the surface, of vigorous convective circulation of groundwater in the uppermost parts of the section. These parts of the section have, of course, been excluded from data treatment.

The difference between predicted and measured equilibrium temperatures is extremely small. With the exception of Imo River-1, the calibrations of the wells produced standard deviations of the temperature residuals of less than 1 °C (Table II). The mean of the standard deviations of the temperature residuals for all wells in all calibrations, except for Haarle-1, are also less than 1 °C. These uncertainties are within the accuracy and precision of the temperature-survey data.

Table IV summarises the statistical analysis. Row 4 presents the mean heat flow (BU) for all columns of table II. The standard deviations of these overall means are small: less than 0.07 BU. In rows 5 and 6 these mean values are compared with relative heat flows obtained for each well using its own regression-calibration data (row 5) and using the calibration of Bolderij-1 only (row 6). The similarity between overall means and those obtained using Bolderij-1 calibration is very close, differences being equal to or less than 0.016 BU in all cases. With the exception of Haarle-1, all three rows agree to within 0.08 BU. Considering the possible sources of errors in measurement, these are very close agreements.

Estimated surface-temperature data are summarized in the same way in table V. Here again the overall mean values for T_0, the individual regression data, and the values from the Bolderij-1 calibration are very close, within a few °C (maximum 5.6 °C).

Equations (7) and (8) are clearly applicable to a wide range of lithologies and capable of estimating consistent relative heat-flow values, regardless of which well data are used for calibrating the coefficients A and c. This is strong evidence of the validity of equations (7) and (8).

We also attempted to combine all the temperature and travel-time data in an overall regression analysis to obtain 'global' estimates for the constants A and c, relative to Bol-

Table IV
Summary of cross-correlation study for relative heat flow (BU) and global calibration schemes, with standard deviations. q* values calculated using A and c from thc global calibrations have been converted into BU by multiplication of calculated relative heat flows by: 0.97346, 0.99830, 1.00369 for the three respective calibrations.

						Country	Netherlands			Nigeria				Oman		U.S.A.	Means of std. devs. temp. residuals in calibration rows	Means of std. devs. heat-flow in calibration rows	
						Well	Bolderij-1	Ten Post-1	Haarle-1	Nun River-1	Ogini-1	Imo River-1	Ebubu-5	Al Huwaisah-1	Saih Rawl-2	Shell Chapman-1			
Calibration data from:-	Number of data points	A = a.q*	Std. dev. A	C = b/a	Std. dev. C	Std. dev. temperature residuals													
Global calibration: q^*, T_o from individual regressions for each well	59	1.00770	0.07093	80.5163	8.3713	5.862	1.000 0.007	1.029 0.013	1.008 0.022	0.512 0.010	0.576 0.003	0.543 0.024	0.427 0.007	0.709 0.017	0.591 0.008	0.622 0.008	0.683	0.012	1
Global calibration: q^*, T_o from means of all calibrations for each well	59	1.03705	0.01004	80.2573	1.447	0.876	1.000 0.007	1.029 0.013	1.008 0.022	0.512 0.010	0.576 0.003	0.543 0.024	0.427 0.007	0.709 0.017	0.590 0.008	0.621 0.008	0.683	0.011	2
Global calibration: q^* from means of all calibrations. All Refo. to upper measured (T, t)	49	1.10397	0.03073	72.7816	3.8910	1.148	1.000 0.007	1.029 0.001	1.016 0.020	0.512 0.010	0.574 0.004	0.541 0.025	0.426 0.007	0.705, 0.017	0.586 0.007	0.611 0.009	0.693	0.012	3
Means of all calibrations, std. dev. of means (from Table 2)	-	-	-	92.84389	-	-	1.000 -	1.029 0.013	1.024 0.049	0.512 0.003	0.574 0.017	0.541 0.011	0.426 0.005	0.705 0.029	0.587 0.029	0.616 0.066	0.765	0.014	4
Individual regression calculations	-	-	-	-	-	-	1.000 0.007	1.029 0.013	1.132 0.000	0.510 0.010	0.582 0.002	0.558 0.023	0.421 0.006	0.710 0.017	0.583 0.007	0.693 0.007	0.588	0.009	5
Heat-flow values using Bolderij-1 calibration	-	-	-	-	-	-	1.000 0.007	1.029 0.013	1.008 0.022	0.512 0.010	0.576 0.003	0.543 0.024	0.427 0.007	0.709 0.017	0.590 0.008	0.621 0.008	0.683	0.012	6

Table V
Surface temperature and standard deviations (°C) - Summary of cross-correlation study and 'global' calibrations.

	Country	Netherlands			Nigeria				Oman		U.S.A.	
	Well	Bolderij-1	Ten Post-1	Haarle-1	Nun River-1	Ogini-1	Imo River-1	Ebubu-5	Al Huwaisah-1	Saih Rawl-2	Shell Chapman-1	
Calibration data from:-												
Global calibration: q^*, T_o from individual regressions for each well		- 3.25 0.32	- 3.88 0.56	- 3.58 0.73	- 3.19 0.83	2.45 0.23	9.42 1.88	14.36 0.63	30.06 0.79	32.46 0.47	21.93 0.88	1
Global calibration: q^*, T_o from means of all calibration for each well		-	-	-	-	-	-	-	-	-	-	2
Global calibration: q^* from means of all calibrations for each well. All Refo. to upper measured (T, t)		- 2.49 0.32	- 3.09 0.56	- 3.04 0.66	- 2.35 0.80	3.25 0.26	9.94 1.86	14.74 0.59	30.29 0.78	32.68 0.46	22.75 0.92	3
Means of all calibrations, std. devs. of means (Table 3)		- 2.66 4.94	- 3.28 5.07	- 3.02 3.52	- 2.64 5.42	2.91 5.12	9.82 3.33	14.67 2.47	30.23 1.55	32.62 1.38	22.15 5.28	4
Individual regression calculations		- 3.21 0.32	- 4.13 0.57	3.76 0.00	0.92 0.69	0.76 0.20	4.39 2.13	17.19 0.45	29.98 0.79	32.83 0.46	16.05 0.74	5
Surface temperatures using Bolderij-1 calibration		- 3.21 0.32	- 3.83 0.56	- 3.55 0.72	- 3.14 0.83	2.50 0.23	9.45 1.88	14.38 0.62	30.07 0.79	32.48 0.47	21.98 0.88	6

derij-1. We adopted three methods:

(1) First of all we used the values for q^* and T_o obtained by the individual regression calibrations. These q^*, T_o estimates are the optimum values for the individual wells. We performed the regression analysis with travel time, t_{ij}, q_i^* and T_{o_i} as independent variables (ith well, jth depth level - see Appendix).

The values obtained for A and c are quite close to those for the Bolderij-1 calibration (Table II). These A and c values were then used to calculate q^* and T_o values for each well separately. The results are presented in tables IV and V (row 1). The means of the standard deviations of the temperature residuals and heat flows are less than 1 °C and 0.013 BU, respectively. The relative heat flows do not deviate significantly from those obtained using only the Bolderij-1 calibration.

(2) Secondly, we used the mean T_o per well (row 1 of Table V), and the mean relative heat-flow per well (row 2 of Table IV) as independent variables together with the travel times (as in (1) - see Appendix). A and c values from this analysis are presented in table IV (row 2). We then used these estimates of A and c to calculate q^* and T_o for each well separately (Tables IV and V). The individual standard deviations of q^* are less than 0.025 BU and the standard deviations of the temperature residuals in fitting the calibration data are less than 1 °C. As in (1) the means of the standard deviations of temperature residuals and heat flows are less than 1 °C and 0.013 BU, respectively, and the relative heat-flows hardly differ from

those for the Bolderij-1 calibration.

(3) Our third approach was to remove the constraining influence of the surface-temperature estimate by referring each well to its own uppermost temperature: travel-time measurement (see Appendix). In this case q_i, T_{ij}, t_{ij} and t_{ij}, $1 < j \leq N_i$ depth points were treated as independent variables. Results for relative heat-flow and surface temperature, estimated for each well, using this overall calibration scheme, are reported in tables IV and V (row 3). The agreement of these heat-flow and T_0 values with those obtained by the alternative methods is very close, and the uncertainty estimates are small.

We conclude that equations (7) and (8) are generally applicable to the estimation of relative heat-flow and subsurface temperatures to a high level of precision and accuracy. These equations are appropriate to most of the rock types encountered in sedimentary basins. Although no satisfactory fit could be obtained for salt sequences, this should not be a serious drawback for the use of the relations suggested.

The heat flows in the ten wells investigated preserve their relative values, independent of the calibration data set or method used. Similarly, the surface-temperature estimates are broadly comparable for the different calibrations.

We recommend using the Bolderij-1 calibration data set rather than the ‘global’ treatments, because the temperature survey of Bolderij-1 is the one most accurately measured. Thus, we take A = 1.039 and c = 80.031. However, calibration data from any of the ‘global’ schemes could be used instead, with insignificant differences in heat-flow and surface-temperature estimates, provided the scaling factors (Table IV) are used to adjust the calculated relative heat flows to BU.

The uncertainty estimates using these calibrations are extremely low. The precision of the relative heat-flow (standard deviations) ranges from 0.007 to 0.024 BU (mean 0.012 BU). The standard deviations for surface-temperature estimates range from 0.32 to 1.90 °C (mean 0.72 °C).

Finally, the accuracy of subsurface-temperature prediction is very high. With the exception of Imo River-1 and Shell Chapman-1, the standard deviations of the temperature residuals are less than 1 °C. The overall mean value is less than 0.70 °C. The absolute differences between predicted and measured temperatures are also very small: 90% of the estimated temperatures within 1 °C of the measured value, 75 % within 0.5 °C, and 40% within 0.25 °C. Two graphic examples of the fits obtained are given in figures 1 and 2.

These slight uncertainties show that our empirical model is applicable over the range of temperature, depth and travel time of the calibration (Tables I and II). The high degree of success probably implies that we have empirically accounted for more variation and more variables than explicitly included in our simple basic model (equation (3)). This does not cause any difficulty in the application of our relations proposed, provided the calibration limits are not exceeded. We expect some inaccuracies beyond these limits.

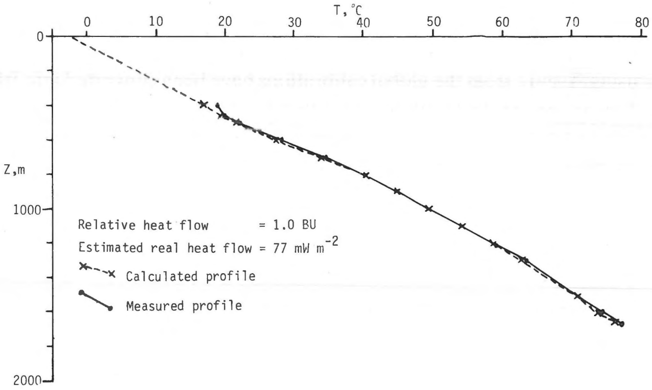


Fig. 1
Measured subsurface temperatures and subsurface temperatures calculated using equations (7) and (10) (Well Bolderij-1, Groningen gasfield).

With suitable temperature data -either equilibrium surveys or time-dependent measurements during logging operations- and accurate well-shoot data, equation (7) can be used to estimate relative heat flow in oil wells.

RELATIVE HEAT-FLOW CALCULATIONS USING SUBSURFACE TEMPERATURES MEASURED DURING LOGGING OPERATIONS

Estimation of relative heat flow

Many modern well data include accurate sound travel-time information based on well-shoot and sonic-log data. Equilibrium-temperature measurements in such wells are, however, rare; in most cases only bottomhole temperatures measured during logging operations are available. The temperature field surrounding a recently drilled hole is disturbed by drilling operations and at the bottom of the hole the temperatures measured are always too low.

EDWARDSON ET AL. (1962) and DOWDLE & COBB (1975) have presented a relation for estimating the undisturbed formation temperature (T_z) from measured bottomhole temperatures (Tm):

$$Tm = T_z + D \ln \left[\frac{\tau_o + \Delta\tau}{\Delta\tau} \right], \tag{9}$$

where T_z = formation temperature before drilling disturbance; Tm = measured bottomhole temperature; τ_o = cooling time (duration of circulation at bottomhole before logging); $\Delta\tau$ = warming-up time (time lapse between end of circulation and measurement of Tm); D = regression coefficient, a constant.

We have used this relation to estimate relative heat-flow values for wells on which travel-time data are available.

For each depth point of which three or more bottomhole-temperature measurements are available, the undisturbed-formation temperature (T_z) and its standard deviation (ST_z) are calculated using a linear regression of equation (9). A Monte Carlo procedure has been used to statistically combine the estimated subsurface temperatures and their standard deviations in the various bottomhole positions in any one well. Ten randomly selected, simulated temperature values were generated from the mean and the standard deviation of the formation temperature in each bottomhole position. Relative heat-flow values (q^*) were calculated from the simulated temperatures using a variant on equation (7):

$$q^* = \frac{1}{t_z} \ln\left[\frac{T_z + c}{T_o + c}\right], \qquad (10)$$

in which T_o is the notional surface temperature at $t_z = O$, corresponding to steady-state heat-flow conditions. The values for q^* calculated in this way are statistically added to those obtained in the previous bottomhole position, yielding the average relative heat flow ($\bar{q}^*$) and its standard deviation ($S\bar{q}^*$) per well. Assuming a normal distribution the 90% confidence limit is obtained by taking the number of degrees of freedom to be the sum of the number of temperatures measured at each depth point minus two, and the number of the depth points minus one. In cases where only two bottomhole temperatures were available at one depth point, only one relative heat-flow value was calculated.

RELATIVE HEAT FLOW IN THE VIKING AND CENTRAL GRABENS OF THE NORTH SEA (U.K. PART ONLY)

As an example, the results are given of relative heat-flow calculations in the central North Sea.

Shell U.K. Ltd. kindly provided data on logging bottomhole temperatures (Tm), cooling and warming-up times (τ_o and $\Delta\tau$), as well as on sound travel-times from the seabed (t_z) of 120 wells in the area.

The relative heat-flow values per well and the associated 90% confidence limits were calculated using a TI 59 calculator.

In this area there is no well with equilibrium temperatures and well-shoot data from which the value of the notional surface temperature (To) can be established.

A lower constraint for To in the North Sea is provided by the Dutch wells Bolderij-1, Ten Post-1 and Haarle-1, which have yielded notional surface temperatures around -3 °C (Table III). This area was exposed to severe permafrost conditions during the last glaciation. For the North Sea area we have adopted a slightly higher notional surface temperature of 0 °C because this area was ice-covered during the last glaciation.

A histogram of the relative heat-flow values obtained is given in figure 3a. The mean relative heat-flow has been found to be 0.60 BU, with a standard deviation of 0.055 BU. There is no systematic relationship between geographical location and the small variations in relative heat-flow found.

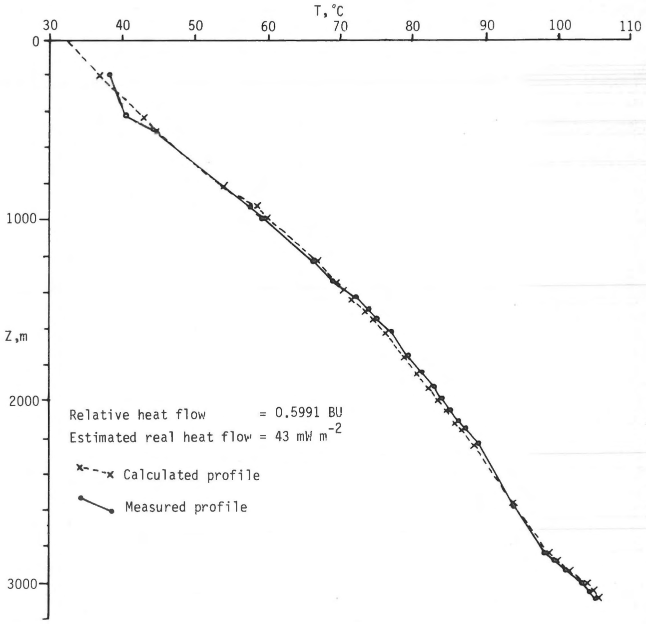


Fig. 2
Measured subsurface temperatures and subsurface temperatures calculated using equations (7) and (10) (Well Saih Rawl 2, Central Oman).

A histogram of the 90% confidence limits of the relative heat-flow values per well is given in figure 3b. It exhibits a distinct mode in the class between 0.01 and 0.02 BU.

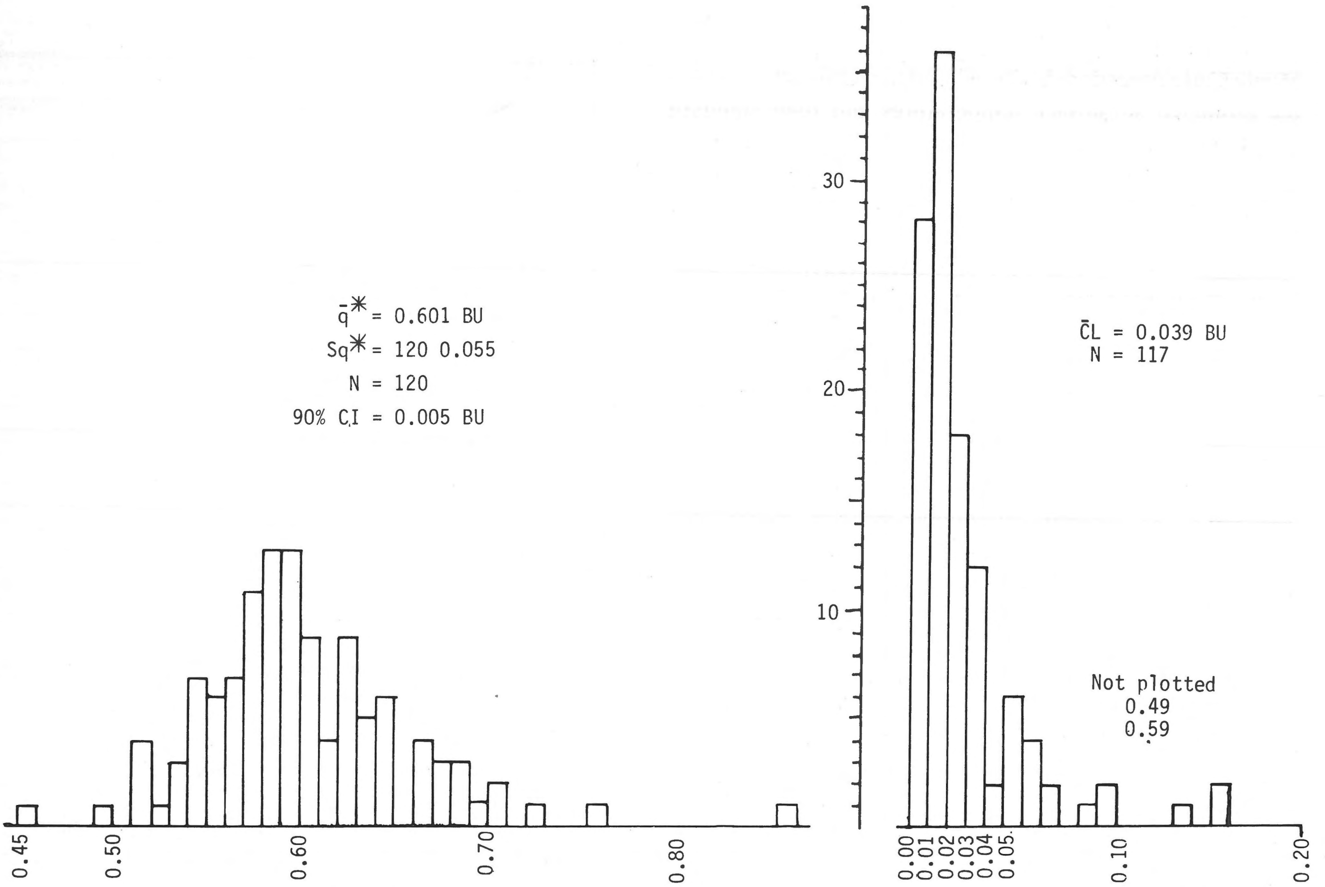
It is not possible to state whether the variation in relative heat flow (standard deviation 0.055 BU) is of geological significance or is due to uncertainties in the measurements and calculations used. It has not been investigated whether this is related to the focussing effects of heat flow through structures.

ESTIMATION OF THE SI VALUE OF THE BOLDERIJ UNIT

The accuracy and precision of the relative heat-flow estimates obtained from oil wells by the method described is satisfactory for the comparison of heat flow in oil provinces. For comparison with heat-flow values, obtained by more conventional means, it is essential to know the value of a Bolderij Unit in SI. Heat flow in SI units has been calculated using published thermal-conductivity data for various rock types, with appropriate corrections for the formation temperature from the wells Bolderij-1 (The Netherlands), Saih-Rawl 2 (Central Oman) and Shell Chapman-1 (Texas) (Table VI).

A general relationship between thermal conductivity and temperature has been suggested by RICHARDSON & POWELL (1976) and RICHARDSON & OXBURGH (1978), using data given by SCHATZ & SIMMONS (1972) and CLARK (1966):

$$K_{T_z} = K_{20}\,\frac{A''}{B'' + C''\,T_z}, \qquad (11)$$



a Heat flow in BU calculated per well

b 90% Confidence limit per well (BU)

Fig. 3
Heat flow in BU calculated per well and 90% confidence limit per well (BU), for wells in the UK part of the North-Sea Viking and Central grabens.

where K_{T_z} and K_{20} are the thermal conductivities at formation temperature and at a room temperature of 20 °C, expressed in K. A″, B″ and C″ are constants for which RICHARDSON & POWELL (1976) give values of 122, 70 and 0.173, respectively.

Substituting equation (10) into Fourier's law (equation (4)), treating the negative sign in the manner described, and integrating between (Z_1 T_1) and (Z_2 T_2) yield

$$q^{SI} = \frac{K_{20}\,A''}{C''\,(Z_2 - Z_1)}\ \ln \frac{B'' + C''\,T_2}{B'' + C''\,T_1}. \tag{12}$$

This relation, in combination with the published ranges for thermal conductivities, has been used to calculate the ranges within which the heat flow in SI units is likely to fall for the above-mentioned three wells. In the case of the well Bolderij-1 the salt sequence provides an additional constraint based on the measured temperature dependence of the thermal conductivity of pure halite. Table VI presents the relevant data, literature sources and the range of the value for one Bolderij unit in SI units as obtained in each separate calculation. The constraints range from 53.5 to 102.0 m W m^{-2} for one BU, but there is a consistent overlapp for all calculations between 74.8 and 79.1 m W m^{-2}. The centre of this overlap is taken as the best estimate of the value of the Bolderij unit

1 BU = 77 m W m^{-2}.

This value, which is open to considerable improvement, is unlikely to be more than 15% in error. It differs considerably from the heat-flow value of 46 m W m^{-2} for the Groningen area given by VAN ENGEN (1975) using thermal gradients in the Rotliegend. This discrepancy is most probably due to abnormal thermal conditions within the Rotliegend gas reservoir.

CONCLUSIONS AND RECOMMENDATIONS

Equations (7) and (10) can be used for calculating heat-flow relative to the heat flow in a standard well to a high level of precision in siliciclastic and carbonate sequences.

These relations are not valid for evaporites. The use of equations (7) and (10) and the Bolderij Unit of heat flow is recommended for the comparison of oil wells or the predic-

Table VI
Estimates of value of Bolderij Unit, the real heat flow at Bolderij-1, based on measured temperature profiles and thermal conductivity measurements culled from literature.

Well	Rock Type	Depth Range m	Temp. °C	Thermal Conductivity Range W m^{-1} K^{-1}	Heat Flow Range mW m^{-2}	Relative Heat Flow BU	Constraints on value of heat flow at Bolderij-1 and Bolderij Unit (BU). mW m^{-2}
BOLDERIJ-1	ZECHSTEIN rock salt	2100 2700	83.8 92.9	5.34 - 7.20[4,6]	74.50 - 100.50	1.0	74.50 - 100.50
Alternative based on pure halite data.	"	"	"	"	71.59 - 96.57	"	71.59 - 96.57
BOLDERIJ-1	CRETACEOUS chalk, marl	900 1500	45.0 67.5	1.68 - 2.20[4,6]	60.31 - 79.13	"	60.31 - 79.13
SAIH RAWL-2 CENTRAL OMAN	JURASSIC AND CRET. limestone	1373 1969	72.5 85.28	1.60 - 2.50[4,6]	32.09 - 50.14	0.5991	53.56 - 83.68
SHELL CHAPMAN 1 TEXAS U.S.A.	Shale, sand (50 : 50)	2361 3463	104.7 138.2	1.67 - 2.31[4 6]	45.50 - 62.03	0.6082	74.81 - 102.00

tion of subsurface temperatures using well-logging and seismic data.

For comparing oil-well data with geothermic observations made by other methods, the value of a Bolderij unit can be taken to be approximately 77 m W m^{-2}. This value is open to improvement but is not likely to be more than 15% in error.

The thermal conductivity of sediments with water-filled pores can be estimated from sonic-log data using a variant of equation (2):

$$K = \frac{V\, q_b}{1.039\,(80.031 + T_Z)},$$

where q_b is the SI value of heat flow in the standard well Bolderij-1.

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Table VII
Listing of symbols used.

a, A b c, C	coefficients
k	thermal conductivity (W $m^{-1}K^{-1}$)
q	Heatflow (mW m^{-2})
q*	Relative heatflow (BU)
T	Temperature (°C)
T_z	Temperature at depth z
T_u	Temperature at top of interval
T_L	Temperature at bottom of interval
T_o	Temperature at surface or seafloor
T_m	Temperature measured at bottom hole
t	One way travel time from surface or seafloor (s)
t_z	One way travel time at depth z
t_u	One way travel time at top of interval
t_L	One way travel time at bottom of interval
V	Sound velocity (m s^{-1})
τ_o	Duration of circulation at bottom hole (s)
$\Delta\tau$	Time lapse since circulation stopped (s)
Z	Depth (m)

APPENDIX: NON-LINEAR LEAST-SQUARES ANALYSIS

We have obtained values for the coefficients, T_o, aq^*, q^*, c etc., in equation (7) by a non-linear least-squares method, minimising,

$$\sum_{j=1}^{N} \phi_j,$$

where

$$\sum_{j=1}^{N} \phi_j = \sum_{j=1}^{N} (T_{j_{observed}} - T_{j_{predicted}})^2. \qquad (I.1)$$

$T_{Jpredicted}$ being the temperature predicted by equation (7), and $T_{Jobserved}$ the measured temperature at the jth depth level. The following presents details of the different analytical methods; the notation follows that of the text.

(1) To obtain To, $A = aq^*$ and c, we impose the minimising conditions

$$\frac{\delta \sum_{j=1}^{N} \phi}{\delta T_o} = \frac{\delta \sum_{j=1}^{N} \phi}{\delta A} = \frac{\delta \sum_{j=1}^{N} \phi}{\delta c} = 0.$$

As it is difficult to solve the non-linear equations obtained under these conditions, an iterative procedure has been used. Since T_o, A and c are known approximately, we can substitute initial guesses for these coefficients into equation (7), T'_o, A' and c', together with the unknown departures of these initial guesses from the 'true' values, E_{T_o}, E_A and E_c. Thus,

$$\begin{aligned} T_o &= T'_o + E_{T_o} \\ A &= A' + E_A \qquad (I.2) \\ c &= c' + E_c \end{aligned}$$

Substituting (I.2) into (7) for the jth depth point (T_j, t_j) and $t_o = o$ at the surface

$$T = (T'_o + E_{T_o})\, e^{(A + E_A)tj} + (c' + E_c)\,(e^{(A + E^A)tj} - 1). \qquad (I.3)$$

Setting

$$T' = T'_o\, e^{A'tj} + c'\,(e^{A'tj} - 1), \qquad (I.4)$$

where

$$T' = (t_o - E_{T_o})\, e^{(A - E_A)tj} + (c - E_c)\,(e^{(A - E_A)tj} - 1), \qquad (I.5)$$

and applying Taylor's formula

$$T = T' + E_{T_o} \frac{\delta T'}{\delta T_o} + E_A \frac{\delta T'}{\delta A} + E_c \frac{\delta T'}{\delta c}, \qquad (I.6)$$

where, from (I.5)

$$\frac{\delta T'}{\delta T_o} = e^{A'tj}$$

$$\frac{\delta T'}{\delta A} = T'_o + c'\, tj\, e^{A'tj}$$

$$\frac{\delta T'}{\delta c} = e^{A'tj} - 1.$$

Equation (I.6) is linear in the unknowns E_{T_o}, E_A and E_c, which are determined by linear least-squares methods. These values E_{To}, E_A and E_c are then used to improve the estimates of T'_o, A' and c' using (I.2). These improved values are used to obtain new values for E_{T_o}, E_A and E_c. This iterative procedure is repeated until the values of E_{T_o}, E_A and E_c are reduced to within a preselected tolerance.

To obtain T_o and A values or T_o and q^* values, a similar procedure is adopted, except that the term $E_c \frac{\delta T'}{\delta c}$ does not appear, since c is now a predetermined constant, and only two coefficients have to be determined.

(2) Treatment of the combined data is very similar to the above procedure, except that, instead of one independent variable t_j, there are three or four of them.

For (a) and (b), we solve A and c with q_i, T_{o_i}, t_{ij} as independent variables (jth depth point, ith well).

For (c), we solve q_0, T_{i_l}, t_{il} and t_{ij} as independent variables; $1 < j \leqslant N_1$ (jth depth point, ith well).

The solutions for (2) were obtained using a FORTRAN-V program. Solutions for (1) were obtained using a FORTRAN-V and a TI-59 calculator program. Duplicate analyses by both procedures produced identical results.