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THE STREAM NETWORK IN THE NETHERLANDS AS A GROUNDWATER DISCHARGE PHENOMENON

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ABSTRACT

Vries, J.J. de (1977). The stream network in The Netherlands as a groundwater discharge phenomenon. Geol. Mijnbouw, 56, p. 103-122.

The stream network in the higher, Pleistocene part of The Netherlands is genetically coupled with groundwater discharge systems of various extents. Streams of a given order can be described as the outcrops of groundwater flow systems of a corresponding order.

The drainage system is controlled by the precipitation surplus (climate), by the resistance of the subsurface to groundwater flow (geology) and by the previous relief and the depth of incision (topography). This concept is defined as the Groundwater Outcrop-Erosion Model (GOEM).

On the basis of this model stream nets can be synthesized theoretically. In this context use is made of data on geology, geomorphology and climate applying groundwater flow formulae and Horton's law of stream order versus stream density. Comparison of the actual and "synthetic" stream-net characteristics shows reasonable agreement.

INTRODUCTION

This paper forms a compilation of part of a PhD-thesis by de Vries (1974); (see also de Vries, (1976). The purpose of this thesis was to explain the present hydrological stage of The Netherlands as a result of its development within the general geological and climatological history of the Quaternary, apart from younger man-made interferences. It also elucidated the functional relations between the present drainage system and the climatological, geological and geomorphological conditions in The Netherlands.

Part I of this paper describes the conceptual model of stream evolution in the Netherlands' Pleistocene area from the Pleistocene to Recent. The model is called the Groundwater Outcrop-Erosion Model (GOEM-concept).

In Part II the functional relations between the drainage network and the geological, geomorphological and climatological data are deduced, for which groundwater flow formulae and Horton's law of stream order versus stream density were applied.

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I: CONCEPTUAL MODEL

The groundwater outcrop-erosion model (GOEM-concept)

The GOEM-concept refers to stream development in areas where the infiltration capacity is rarely exceeded. Here all surplus precipitation eventually percolates into the subsoil and becomes part of a groundwater flow system before it reappears in a surface channel system which acts as a groundwater drainage network. This condition is found in relatively flat areas with a high subsurface permeability and moderate precipitation rates. Due to this low relief, exceedance of the infiltration capacity leads to temporary water storage at the surface, rather than to surface runoff. Under such conditions a stream channel can be described as a groundwater drain and a linear groundwater discharge phenomenon. Thus stream channels of different orders are considered simultaneously as outcrops of groundwater flow systems of corresponding orders.

The GOEM-concept is in contrast with the well-known infiltration-model of surface erosion and discharge as presented by Horton (1945). In the Horton-model erosion by surface runoff occurs when the infiltration capacity is exceeded and the potential for erosion surpasses the resistance to erosion. The resulting drainage system is therefore controlled by the climate, the topography and the erodibility of the upper soil layers. In the GOEM-concept, however, the case where the infiltration capacity is exceeded can be neglected and the surface drainage system is controlled by the climate, the topography and the resistance of the subsurface to groundwater flow.

According to Kirkby & Chorly (1967), the Horton-model is "one end-member of a wide spectrum of erosion models; the other end-member applies to slopes with high infiltration capacity and thick soil covers where throughflow dominates and overland flow, with its attendant channel initation, only occurs in a few restricted areas".

The "throughflow-model" however does not as yet, form the end-member of the series of models because it still regularly shows overland flow in its final stage, which is exceptional or non-existent in our GOEM-concept. In the GOEM-concept the soil cover of the throughflow-model is replaced by a permeable aquifer of considerable thickness in a flat accumulation area. This should be considered as the true end-member of the series of models.

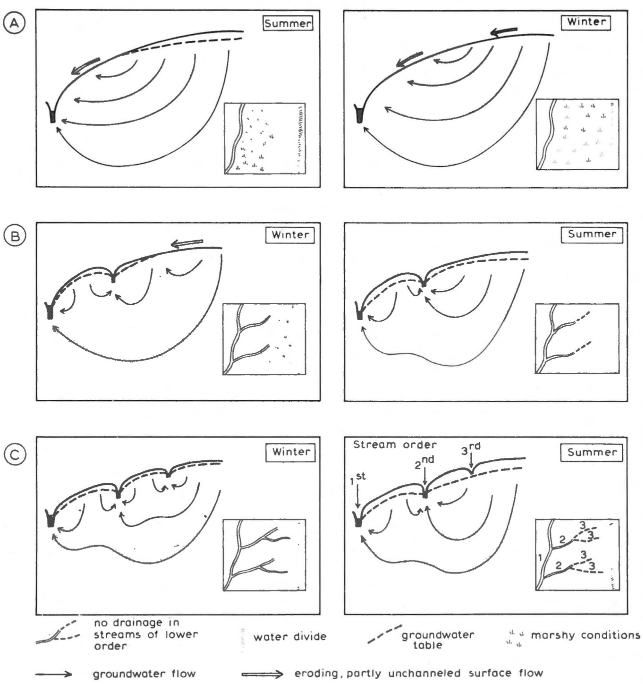


Fig. 1
Stream development according to the Groundwater Outcrop-Erosion Model. (A) represents the initial stage with unchanneled surface runoff and temporary storage in marshes in all seasons; (B) represents an intermediate stage with unchanneled surface runoff exclusively in the wet period; (C) represents the final stage with exclusively groundwater discharge in all seasons. A, B and C refer to consecutive sub-stages within each stage of development of an expanding stream system; thus sub-stage C will first be reached near the discharge base and subsequently shift towards the divide.

Stream development according to the GOEM-concept – The stages in the development of a stream network according to the GOEM-concept are shown in fig. 1.

When the initially low groundwater discharge capacity is exceeded by the precipitation surplus an enlargement of the groundwater discharge area results (fig. 1A). In this groundwater-outcrop zone erosion is started by a lack of possible infiltration with a simultaneous excess of surface water, and by outflowing groundwater. Upward seepage and outflow of groundwater cause a reduction of the granular pressure and stimulate erodibility by surface runoff.

Normally there are some inherited features which will influence subsequent development of the stream network. In The Netherlands a widely-spaced system of rivers was present at the beginning of the Holocene (fig. 1A), which was inherited from the previous Pleistocene (see section 2). From this primary stream system a secondary, more dense stream system developed by headward migration of the erosion in equilibrium with the Holocene climatological conditions.

Initially widely-spaced and relatively deeply eroded streams developed in equilibrium with low precipitation rates of high frequency (fig. 1B). That meant that surface runoff stopped during periods with a high-frequency precipitation surplus. Subsequently a denser stream net with less-deeply eroded streams of higher order was formed. This was caused by headward migration of erosion under higher precipitation rates, which occurred with lower frequencies (fig. 1C). As a matter of fact, the stream distribution of stage C represents a growth model: it expanded from the erosion base towards the divide. Thus fig. 1A, B and C represent the substages within each stage of an expanding system. Each successive drainage network is geometrically similar to the preceding one (Leopold *et al.* 1964, p. 420; Morisawa, 1964).

In this way a drainage network was created which can be characterized as follows (figs. 2 and 3):

(1) – Within an existing and expanding drainage system, the stream network organizes itself in such a way that it complies with Horton's logarithmic law of stream order and stream density. This means that streams of different order provide a hierarchy of channel size and stream density, so that streams of the lowest order show the largest channel sizes, the deepest incision and the lowest stream density. Thus any drainage system of a given order is characterized by channels eroded to a corresponding depth. An increase in recharge combined with a rise of the groundwater table, causes an increase in the number of streams which contributes to the groundwater discharge (fig. 2 and 3a). This implies that the density of that part of the stream net which contributes to the discharge increases. Thus the stream spacing and the associated drainage resistance[2)] descrease as a function of a rising groundwater table (fig. 3c). As a result the groundwater discharge

[2)] The specific drainage resistance is defined as the ratio between the hydraulic head difference between the divide and the stream on one side, and the groundwater discharge to the stream on the other. It is a function of the transmissivity as well as the stream spacing.

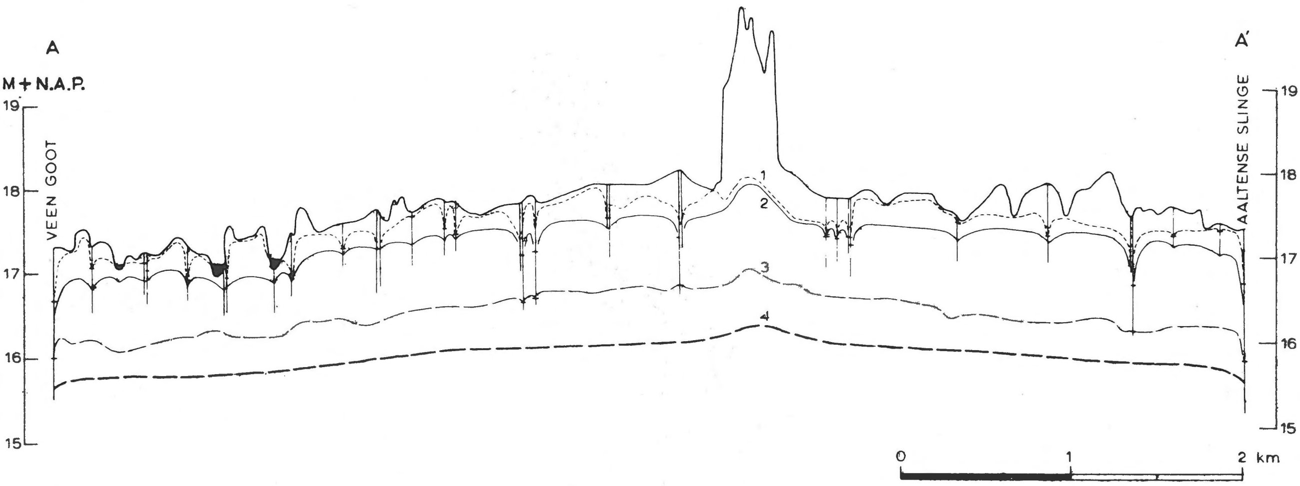
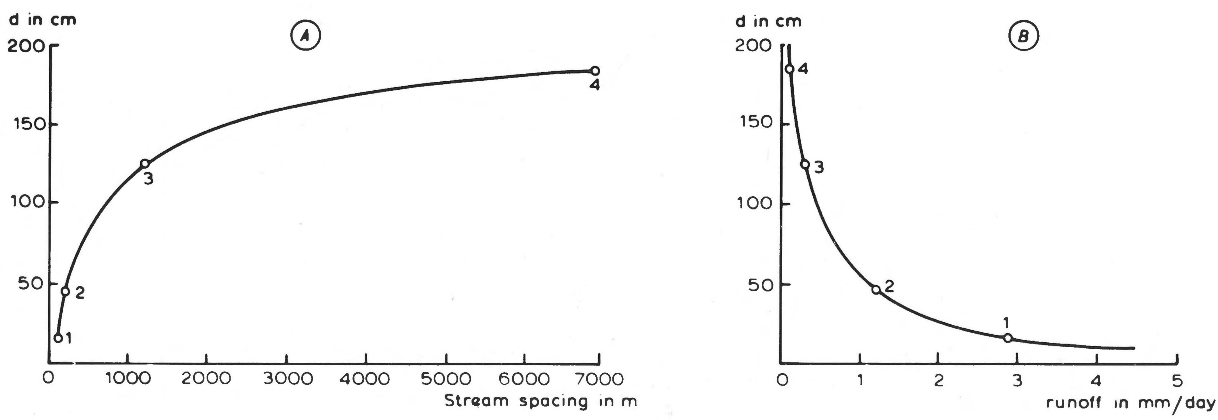


Fig. 2
Section between the small rivers Aaltense Slinge and Veengoot (East Gelderland), showing groundwater tables for different conditions and the corresponding water levels in the channels. The curves 1, 2, 3 and 4 respectively refer to: winter, wet period (1); winter, average rainfall condition (2); normal summer, only larger streams participate in groundwater discharge (3); extreme dry summer, all streams dry (4). (After Ernst, de Ridder & de Vries, 1970).



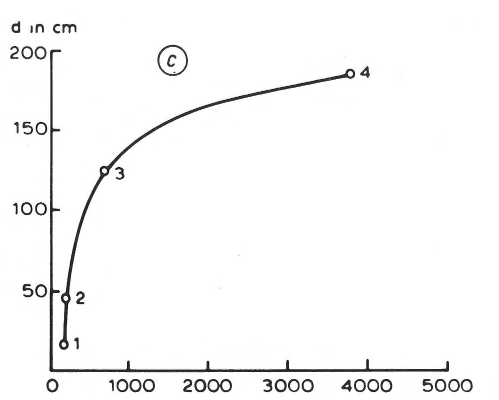


Fig. 3
Relations between groundwater depth d (in cm below soil surface) and stream spacing, drainage resistance and runoff for East Gelderland. The numbers 1, 2, 3 and 4 refer to the groundwater tables in fig. 2; runoff means the average groundwater discharge of the streams. (After Ernst, de Ridder & de Vries, 1970).

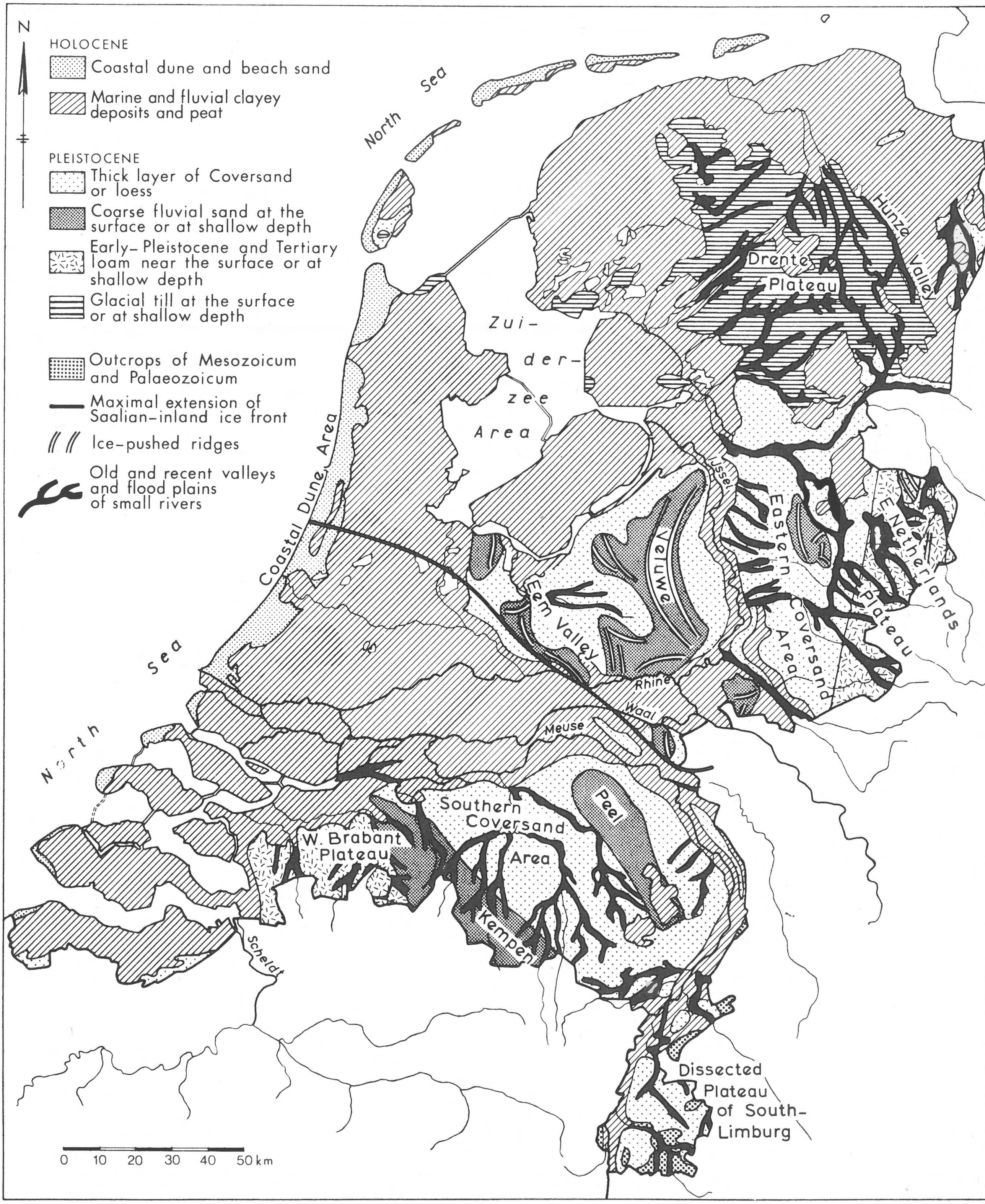


Fig. 4
Geomorphological-geological map derived from the National Soil Map and the Geologic Map of The Netherlands. Raised bogs in the northeastern and southern districts have largely disappeared because of the need for fuel.

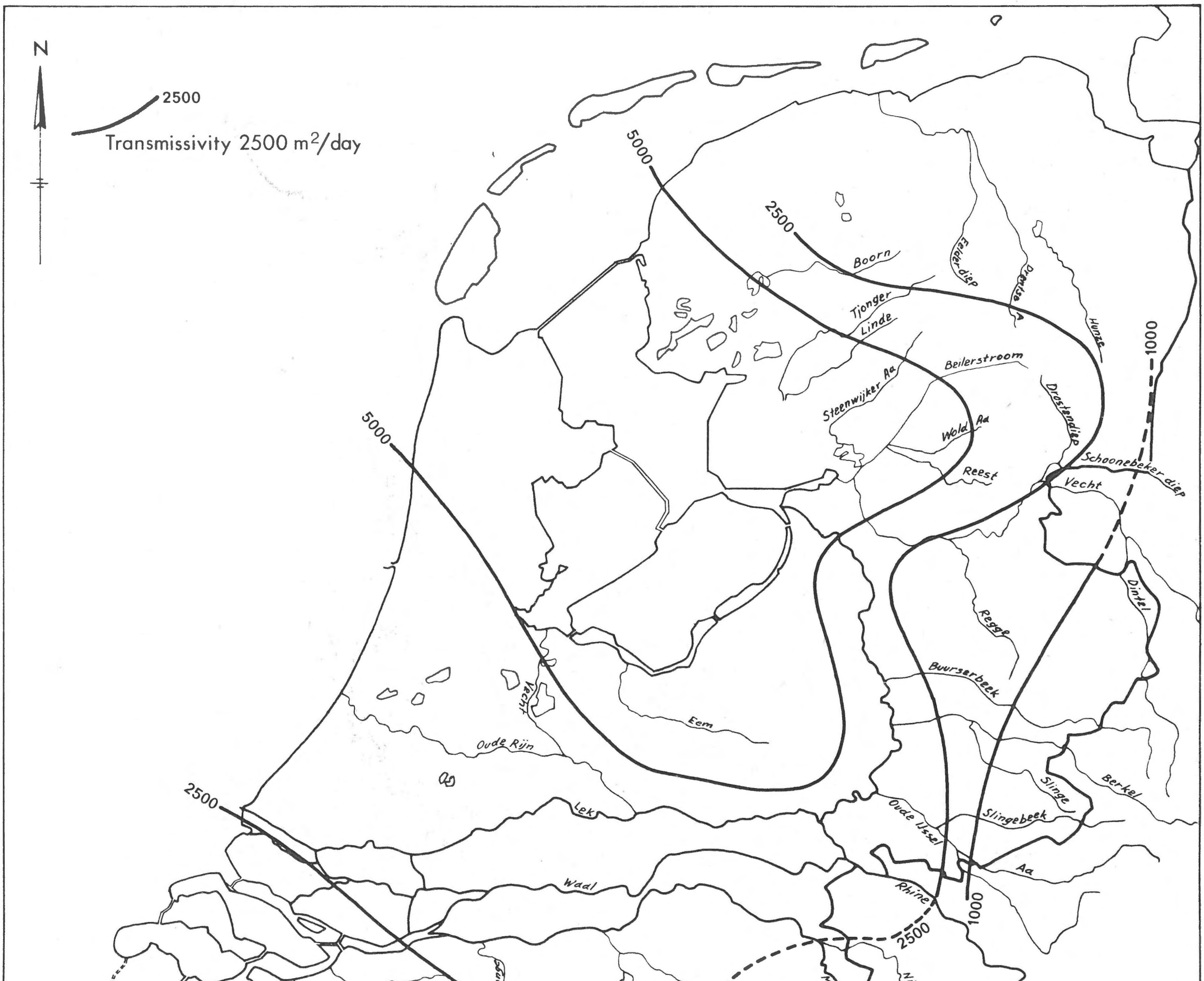


Fig. 5
Map showing the transmissivity of that part of the Pleistocene aquifer which participates in the discharge of the precipitation surplus.

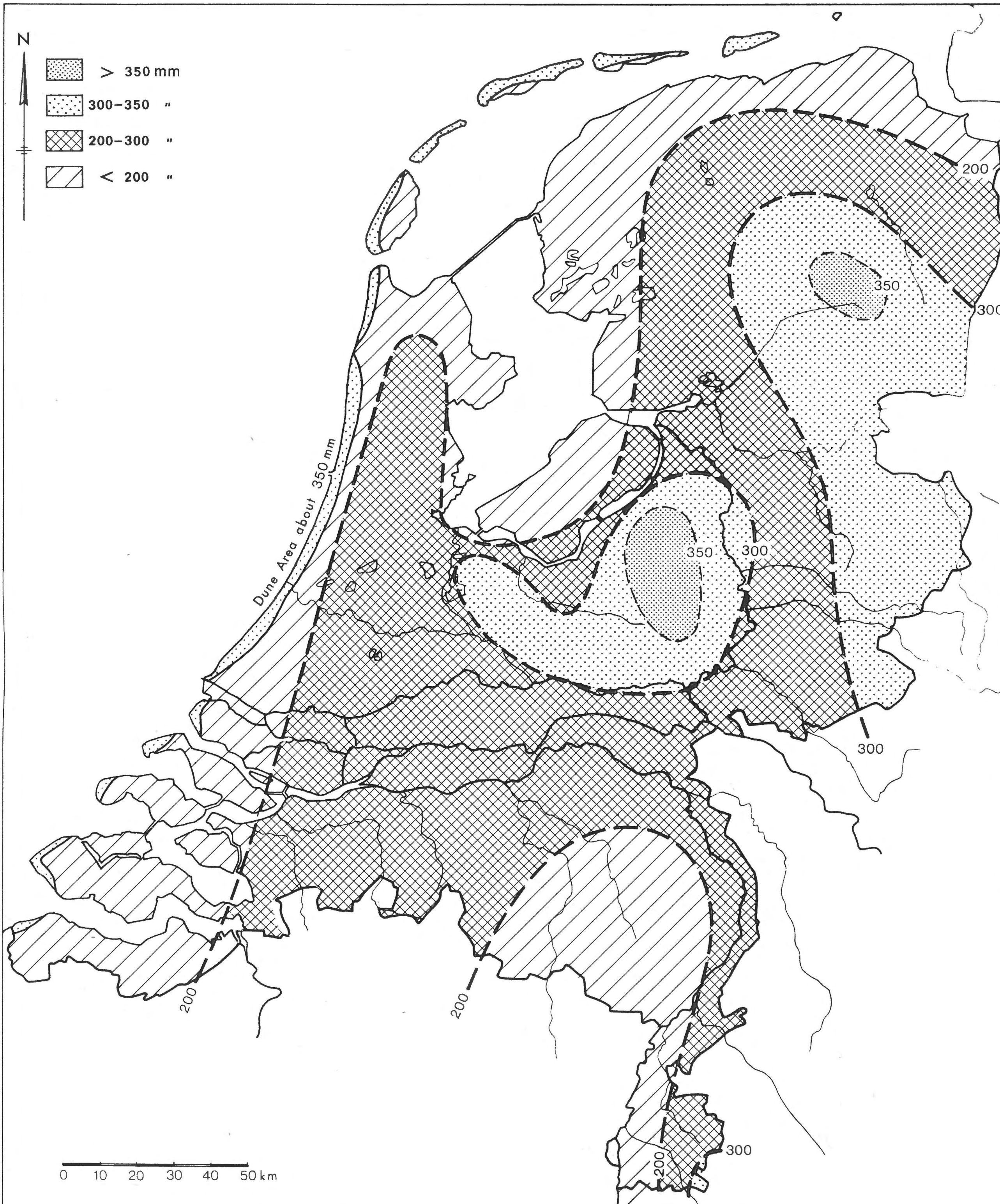


Fig. 6
Areal distribution of the annual precipitation surplus in mm.

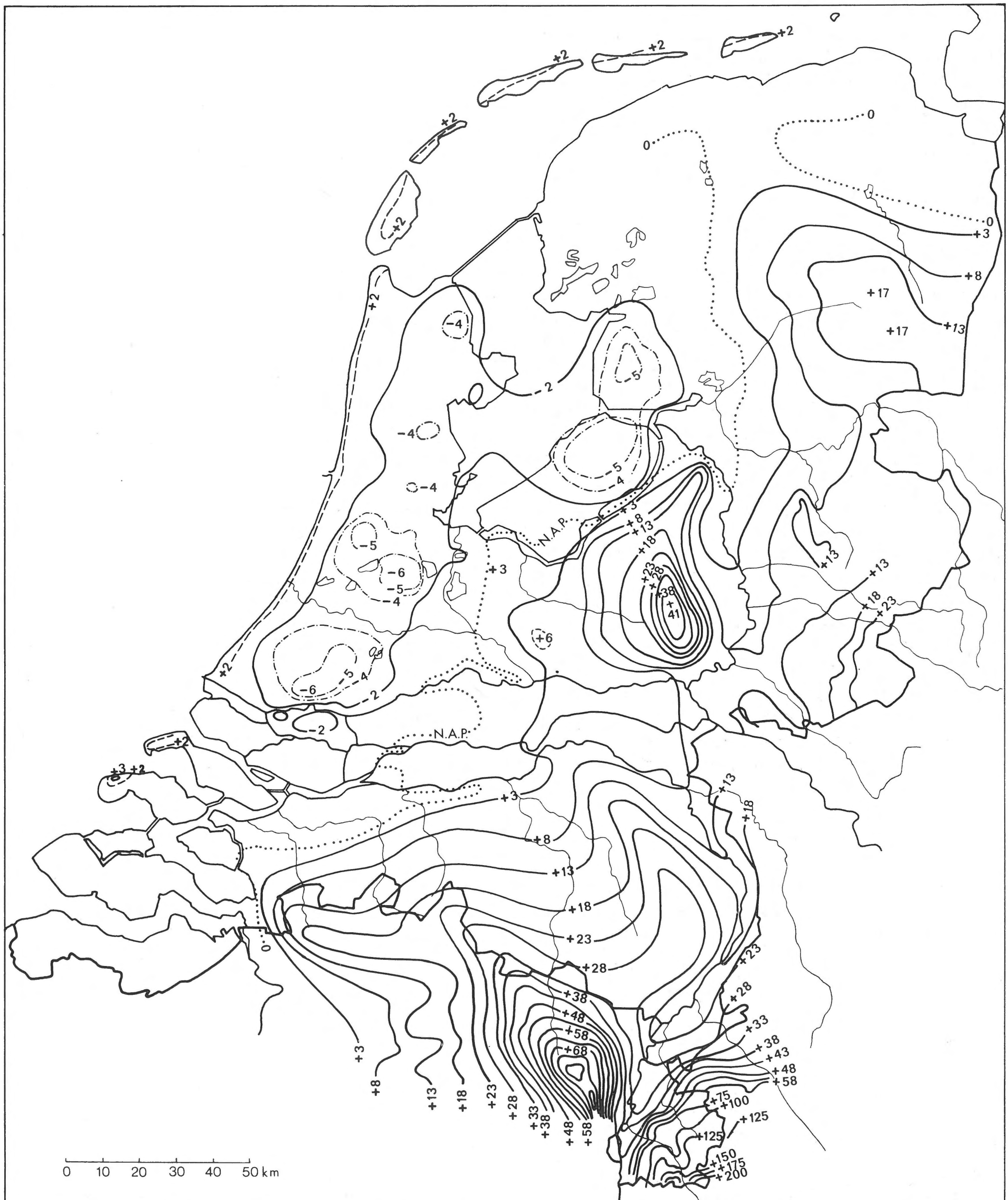


Fig. 7
Contour map of the average hydraulic head of the Pleistocene aquifer relative to sea level, representing the hydraulic gradients of the primary groundwater flow systems.

will increase (fig. 3b).

(2) – The groundwater discharge capacities of the drainage systems of a certain order correspond to certain recharge rates. The system with the smallest and least eroded stream channels shows the highest rainfall rates. The various rainfall rates can be characterized by their frequency of occurrence.

The GOEM-concept and stream development in The Netherlands

Physiographical outline of the Pleistocene area – The Pleistocene area of The Netherlands covers the central, eastern and southern parts of the country (fig. 4). This area can roughly be characterized as a Pleistocene fluvial fan with a thickness increasing from 50 m in the south and east to over 200 m in the north and west. The predominantly coarse fluvial sandy deposits are covered by fine eolian sands (so called coversands) with an average thickness of 5 to 10 m and a maximum thickness of 20 m. The coversands form an undulating topography with maximum height differences of 2 m. They were deposited under a kind of arctic desert condition during the Saalian and Weichselian. Loamy deposits on top of the coarse fluvial sediments are found on two plateau-shaped areas: the Drente Plateau and the West Northbrabant Plateau. Extensive raised bogs developed on these plateaus during the Holocene (fig. 4).

For a fuller description of the geological situation the reader is referred to Rijks Geologische Dienst, 1975.

The average permeability factor of the fluvial sediments amounts to about 25 m^2/day. Fig. 5 shows a transmissivity map of this aquifer. The coversand shows a permeability of a few meters per day. Loamy and peaty layers of much lower permeability are irregularly distributed within the coversand.

The Pleistocene area can be considered morphologically as a slightly north-westward sloping plain. To the west and north this plain dips under sub-horizontal Holocene clay and peat layers of the coastal polder area. The average regional slope of the Pleistocene surface amounts to about 1 : 2000. The gradients to the small rivers, developed perpendicularly to the regional slope are only a little steeper than the regional slope. The slightly undulating coversand topography with height differences of up to a few meters covers this fluvial topography. The presence of some ice-pushed ridges and related deeply scoured glacial tongue basins is a remarkable feature in the Pleistocene area. The slopes of the ridges amount to 1 : 100.

The present climate in The Netherlands is semi-humid and maritime with moderate temperatures and an average annual precipitation of about 750 mm in the Pleistocene area. The annual evaporation there is approximately 450 mm. During the summer half-year the evaporation and the precipitation are counterbalanced. During the winter the precipitation surplus amounts to 300 mm (fig. 6).

The infiltration capacity in The Netherlands is rarely exeeded because of the high permeability of the subsurface, the moderate relief and the moderate precipitation rates. Whenever the infiltration capacity is exceeded, temporary water storage at the surface is caused. This water ultimately infiltrates and so contributes to the groundwater flow system. The moderate surface slopes and the small incision depths of the rivers have resulted in a shallow groundwater table (fig. 7). Practically everywhere the groundwater table is found within two meters below the surface. The ice-pushed ridges form an exception to this rule; there the phreatic surface may reach 30-40 m below the surface.

Table 1.
Stratigraphical-lithological table of the eastern and southern parts of The Netherlands.

Chronology	Climate	Main deposits	max. thickness
Holocene	moderately warm	peat and brook deposits	10 m
Weichselian	(sub)-arctic	eolian sand (coversand) coarse fluvial deposits	20 m
Eemian	moderately warm	fine, silty sand and peat	5 m
Saalian	(sub)-arctic, land-ice in the northern part of The Netherlands	glacial till (Drente Plateau) eolian sand glacio-fluvial sand and clay	100 m
Older Pleistocene periods	alternately cold and warm	fluvial coarse sands with clay layers intercalated (outcropping on the West Northbrabant Plateau)	150 m
Tertiary	warm	mainly marine clayey deposits (lower base of the aquifer)	

Stream development – At the end of the Pleistocene (Weichselian) a periglacial climate prevailed in The Netherlands with an arctic desert character. The discharge capacity of the pre-Weichselian valleys was too large for the new climatic conditions, and the discharge too small to keep the valleys free from eolian sand. In consequence many valleys were buried or dammed, and the drainage system desintegrated. An exception to this pattern was formed by the stream system of the major rivers which originated outside The Netherlands and continued to flow from the Weichselian up to the present, (e.g. van den Toorn, 1962; Riezebos & Slotboom, 1970).

After the Pleistocene the increase in precipitation resulted in an excess of the precipitation surplus over the groundwater discharge capacity of the primary drainage system (see stage A of fig. 1). Initially this excess caused large scale marshy conditions, especially in the depressions and on the broad divides of the undulating coversand topography. During the Holocene part of the rainfall surplus which was not discharged by the primary groundwater flow systems, caused erosion of the secondary drainage system (see stages B and C of fig. 1). In this way headward erosion gradually, recovered the stream system and created an increase in drainage density and a larger capacity for specific groundwater discharge. This meant a shift from direct surface runoff and retention storage in marshes, to infiltration and surface runoff in channels fed by groundwater flow systems of various orders.

The plateau-shaped areas with less pervious surface deposits form an exception to this general pattern of stream development. The relatively low infiltration capacity resulted in permanent surface retention and ponding. Erosion by surface runoff was prevented by a lack of sufficient relief, an irregular eolian topography and resistant clay layers near the surface. This caused the development of local peat bogs which ultimately evolved into extensive raised bogs. Generally speaking, a well-defined stream system developed producing a secondary topography in areas with a sloping surface; in the plateau areas the development of the secondary topography resulted in marshy conditions. Marshes also developed in areas with a concave topography.

On the ice-pushed ridges no secondary drainage system developed. Their primary relief was large enough to create the hydraulic gradients required to discharge the rainfall surplus as groundwater. The present-day dry valleys on the ridges were eroded during the Weichselian period under periglacial conditions. At that time the transmissivity of the subsurface was reduced by permafrost.

II: APPLICATION OF THE MODEL

The synthesis of the stream network in The Netherlands' Pleistocene area from the independent variables has been accomplished in the following way:

– Each stream system of a given order has been adjusted so as to be able to discharge, by the intermediary of a groundwater flow system of a given order, a precipitation surplus occurring with a certain probability.

– The spectrum of precipitation rates between zero and the maximum rainfall rate leads us to expect an almost infinite differentiation in stream order. Such an infinite differentiation is most efficient for groundwater discharge, but not for surface discharge. The flow resistances for groundwater flow and channel flow are subjected to an opposing tendency. The lowest flow resistance in the channel system is reached if one channel is of first order and all the remaining ones are of second order. This process follows the principle of transmitting the largest part of the discharge by means of the largest channel. The final stream pattern will be a compromise between the two opposing tendencies and will comply with Horton's logarithmic law of stream density and stream order (e.g. Chorley & Kennedy, 1971, p. 235).

– An approximate correlation between stream order and precipitation probability has been obtained from an overall empirical knowledge of the hydrologic situation in The Netherlands. With the aid of Horton's law a subsequent refinement has been made by an iteration procedure.

Primary groundwater flow systems

The primary groundwater flow systems are formed by the regional groundwater flow towards the Holocene coastal plain. Their hydraulic gradients are indicated in fig. 7.

The mean areal discharge or average flux through the phreatic surface U is defined as:

$$U = \frac{q_o}{L^*} \tag{1}$$

where q_o represents the discharge per unit width at the discharge base and L^* the distance between the divide and the discharge base. More generally we have:

$$U(x) = \frac{dq}{dx} \tag{2}$$

Following Ernst (1962) the specific drainage resistance Υ can be defined as:

$$\Upsilon = \frac{\Delta h}{U} \tag{3}$$

where Δh is the difference in potential energy heigth or the difference in hydraulic head between the divide and the discharge base. In this study Υ is expressed in days.

In the Pleistocene area, the hydraulic gradients are small in comparison with the thickness of the aquifer. Furthermore the Pleistocene fan is rather homogeneous as an aquifer. Therefore it is assumed that the regional groundwater flow is horizontal and parallel in an aquifer of constant thickness, with uniformly distributed recharge.

Two cases can be distinguished: (a) the mean areal discharge $\overline{U}$ equals the average precipitation surplus $\overline{N}$, and (b) $\overline{U}$ is smaller than $\overline{N}$.

(a) – The mean areal discharge $\overline{U}$ equals the average precipitation surplus N. According to Darcy's law.

$$q = Nx = -Kb\frac{dh}{dx} \tag{4}$$

where x = distance to the divide and h = hydraulic head relative to the aquifer's base. Integration yields:

$$\tfrac{1}{2} Nx^2 = Kb\,[h(o) - h(x)] \tag{5}$$

Hence: $$N = \frac{2Kb\Delta h}{L^{*2}} \tag{6}$$

Substitution of eq. (6) in (3) gives:

$$\Upsilon = \frac{L^{*2}}{2Kb} = \Upsilon_1 \tag{7}$$

This implies a parabolic groundwater table (fig. 8A).

(b) – The total discharge is less than the total precipitation surplus, in formula:

$$q_o < NL^* \tag{8}$$

In this case the groundwater table reaches the surface. For the conditions of fig. 8B Darcy's law gives:

$$q_o = UL^* = Kbs^* \tag{9}$$

and: $$NL^+ = q_o = Kbs^* \tag{10}$$

where L* is the distance from the divide to the point of discharge and s* represents the surface slope which is approximately constant. Substitution of eq. (9) in (10) yields:

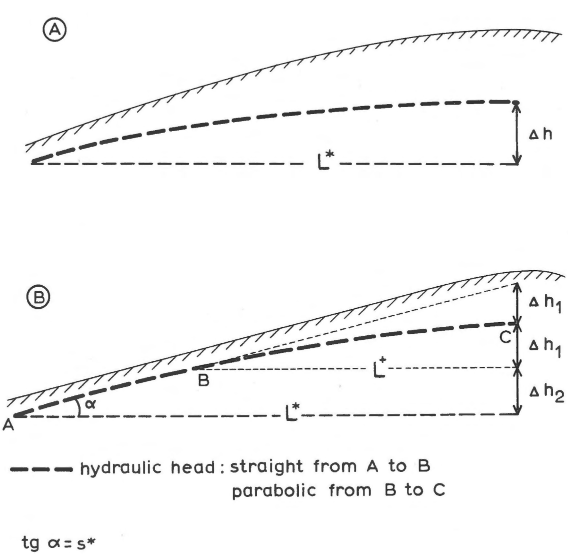


Fig. 8
Models for primary groundwater flow systems.
A – for deep groundwater tables;
B – for shallow groundwater tables.

$$L^+ = \frac{UL^*}{N} = \frac{Kbs^*}{N} \tag{11}$$

From the parabolic shape of the upper part of the groundwater table it follows that the slope $s(L^+)$ of the groundwater table for $x = L^+$ is given by (see fig. 8B):

$$s(L^+) = \frac{2\Delta h_1}{L^+} \tag{12}$$

And: $$s(L^+) = s^* = \frac{\Delta h_2}{L^* - L^+} \tag{13}$$

Thus: $$\frac{\Delta h_2}{L^* - L^+} = \frac{2\Delta h_1}{L^+} = s^* \tag{14}$$

Combination of eq. (14) with (13) yields:

$$\Upsilon = s^* \frac{L^* - \frac{1}{2}L^+}{U} = \Upsilon_2 \tag{15}$$

Substitution of eq. (15) in (9) gives:

$$\Upsilon_2 = \frac{L^*(L^* - \frac{1}{2}L^+)}{Kb} \tag{16}$$

Introduce $$\frac{L^+}{L^*} = \epsilon, \tag{17}$$

then: $$\Upsilon_2(\epsilon) = \frac{L^{*2}}{Kb}(1 - \tfrac{1}{2}\epsilon) \tag{18}$$

Substitution of eq. (17) in (11) yields:

$$\frac{U}{N} = \epsilon = \frac{Kbs^*}{NL^*} \tag{19}$$

From eq. (7) and (18):

$$\frac{\Upsilon_2(\epsilon)}{\Upsilon_1} = 2 - \epsilon \tag{20}$$

Obviously, for $\epsilon = 1$, eq. (18) reduces to eq. (7).

Apart from the ice-pushed ridges, the slope of the primary topography of the Pleistocene landscape is very small. As a result the mean areal discharge U is much less than the precipitation surplus N.

$U \ll N$ implies that (see eq. (19)) $\epsilon \ll 1$, and (18) reduces to:

$$\Upsilon_2(0) \approx \frac{L^{*2}}{Kb} \tag{21}$$

For the Pleistocene area the following average data apply:

Kb = 2500 m^2/day; Δh = 17 m and $L^* = 3 \times 10^4$ m. Substitution of these values in (19) and (20) yields:

$\epsilon = 0.07$ and $\Upsilon(\epsilon) = 3 \times 10^5$ day, and thus U = 0.05 mm/day. Since the average precipitation surplus amounts to 0.85 mm/day, nearly all the precipitation surplus will be discharged by the secondary drainage system. The ice-pushed ridges form an exception. The most extensive ridge, the East Veluwe hill, shows the following characteristics:

Kb = 4500 m^2/day; s^*L^* = 70 m; $L^* = 1.6 \times 10^4$ m. With a precipitation surplus of 1 mm/day it is evident that $N = U$, $\epsilon = 1$ and $\Upsilon = \Upsilon_1 = 3 \times 10^4$ day.

The ice-pushed ridges show deep groundwater tables. This is contrary to the normal conditions in the Pleistocene area. The groundwater depth (d) reads:

$$d = s^*L^* - N\Upsilon_1$$

Substituting the Veluwe data yields: d = 40 m.

Secondary drainage system of first and second order

The secondary drainage system constitutes the groundwater flow towards the smaller streams; it is superimposed on the regional, primary system.

In the following, streams are classified according to their order of magnitude, in such a way that the channels of the lowest order possess the largest geometrical dimensions.

The secondary groundwater flow systems of first and second order can be visualized as a flow system trending towards a parallel set of streams. If extensive aquitards are absent near the surface, the groundwater flow resistance Υ of these systems can be described by the formula given by Ernst, 1962:

$$\frac{\Delta h}{U} = \Upsilon = \frac{L^2}{8Kb} + L\,\Omega \qquad (22)$$

where: L = spacing between parallel streams;
Kb = transmissivity of the aquifer;
$L\Omega$ = radial flow resistance (explained below).

The first term of eq. (22) corresponds to eq. (7) for $L = 2L^*$; the second term represents an extra flow resistance caused by the upward bending and contraction of the flow lines near the channel. This radial flow resistance depends on the shape of the stream channel and on the ratios between the hydraulic soil properties of the fine coversand layer and those of the coarse Pleistocene aquifer.

The ratios of the soil properties in the Pleistocene area are on average:

$K' : K = 1 : (5 \text{ to } 10)$
$b' : b = 1 : (5 \text{ to } 25)$

The symbols K' and b' refer to the coversand layer.

Before stream regulation was carried out, the streams flowed in broad and shallow valleys. During the winter the depth of the stream channel was much smaller than its width, so that the wet perimeter B of a stream channel was smaller than the thickness b' of the coversand layer. With these relations and some modified formulas on radial flow given by Ernst, the following formula can be derived (Ernst, 1962: eqs. (41), (47), (53) and fig. 14):

$$\Omega = \frac{1}{\pi K'} \ln \frac{5b'}{B} \qquad (23)$$

The following relation is obtained after substitution for values of B between 5 and 10 m, and values of b' between 5 and 20 m:

$$K'\Omega = 0.3 \text{ to } 1.0 \qquad (24)$$

During the winter, the elevation of groundwater tables near the surface causes the mean slope of the groundwater table to follow approximately the general slope of the surface topography from the divide to the stream channel. If ρ denotes the mean surface slope, then:

$$\rho = \frac{2\Delta h}{L} \qquad (25)$$

Combining eq. (25) and (22) yields:

$$\frac{\rho}{U} = \frac{L}{4Kb} + 2\Omega \qquad (26)$$

Substitution of eq. (23) gives:

$$\frac{\rho}{U} = \frac{2}{\pi K'} \ln \frac{5b'}{B} + \frac{L}{4Kb} \qquad (27)$$

The ratio between the two right hand terms of eq. (27) changes gradually during the development of the drainage system. At the end of the Pleistocene the large rivers of zero order were the only ones functioning in the discharge process. At that time the following conditions held:

- large stream spacing: $L \approx 10$ km;
- diffuse groundwater discharge;
- large wet perimeter of channels;
- small radial flow resistance: $L\Omega \to 0$.

During the development of the stream system in the Holocene, the stream spacing L decreased and the groundwater discharge became concentrated towards eroding channels. Simultaneously the radial flow resistance increased, which in turn resulted in a smaller L than would otherwise have been necessary. The relations between L, B and U in eq. (27) will be discussed.

The stream channels of zero order are considered to be a basic topographic element. The groundwater discharge U_o of the drainage system of zero order can be computed by eq. (26) for $\Omega \to 0$, from its stream spacing L_o, the transmissivity of the Pleistocene aquifer Kb and the relief factor ρ. This discharge amounts to $U_o = 0.4$ mm/day for average conditions of the large groundwater flow systems in the Pleistocene area, characterized by $L = L_o = 10$ km, $\rho_o = 1 : 2500$ and $Kb = 2500\ m^2$/day. As a first approximation, the assumption is made that the stream system develops initially in equilibrium with the average winter precipitation surplus $\overline{N}_w$. That means that the hydraulic and geometric properties of the drainage system of lower order will be such that the quantity $\overline{N}_w$ is discharged by groundwater flow to the streams of lower order.

The terms of eq. (27) will be analyzed for average conditions, when $\rho = 1 : 2500$, $K = 50$ m/day, $K' = 5$ m/day and a discharge which varies from $U_o = 0.4$ mm/day to $U = \overline{N}_w =$

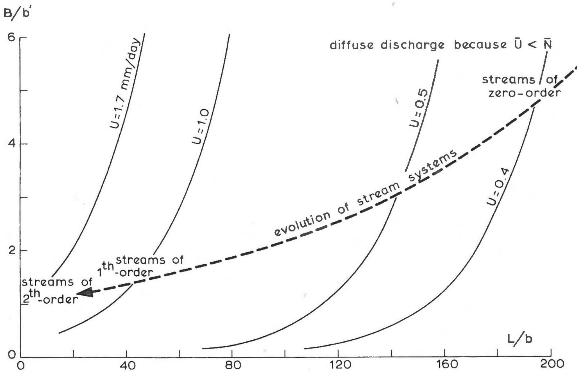


Fig. 9
Relation between B/b′ and L/b for different values of $\bar{U}$. The change in this relationship during the evolution of the stream system is shown.
B = channel width; $\bar{b}$ = thickness of the coversand layer;
L = stream spacing; $\bar{U}$ = average areal discharge.

1.7 mm/day[3]. Substitution of the hydraulic soil properties in (27) yields:

$$\frac{1}{U}\left(\frac{\text{day}}{\text{m}}\right) = \frac{1000}{\pi}\ln\frac{5b'}{B} + \frac{12.5L}{b} \tag{28}$$

In fig. 9 the relation is shown between B/b′ and L/b according to eq. (28). The physical implication of the relation between these quantities is that for B/b′-values smaller than indicated for a given value of L/b′ and U, the radial flow resistance Ω would become too large to satisfy eq. (28). This means that the discharge would become diffuse, thus increasing the wet perimeter B and reducing Ω. The stream spacing reaches a maximum value if the radial flow resistance approaches zero. This occurs if B/b′ = 5. It must be noticed, however, that for a value of B/b′ > 1 eq. (23) will not yield accurate results. For small values of Ω however, the accuracy is not of primary importance. Given the limiting value B/b′ = 5, the wet perimeter reaches a value of 60 m, for average conditions of the Pleistocene area, with b = 50 m, b′ = 10 m and maximum spacing of streams of zero order amounting to 10 km.

For the stream system which is adjusted to the average winter precipitation surplus, eq. (28) yields a stream spacing which varies from 1000 m to 2000 m if B varies from 20 m to 30 m. It is plausible from these computed values and the observed data of actual stream nets of the Pleistocene area, that there will be another stream class between the stream class of zero order and the stream class adjusted to $\bar{N}_w$. As a first approximation it is assumed that the groundwater discharge capacity of this "in-between class" is in equilibrium with the average annual precipitation surplus $\bar{N}$ = 0.85 mm/day. In the following this "in-between class" is denoted as the secondary stream system of first order. The system which is adjusted to $\bar{N}_w$ is denoted as the secondary stream system of second order.

Substitution of the values in m/day of $\bar{N}$ and $\bar{N}_w$ for U in eq. (26) yields the equations for the stream spacing L_1 and L_2 of the drainage systems of first and second order:

$$L_1 = 8Kb(586\rho_1 - \Omega) \tag{29}$$
$$L_2 = 8Kb(293\rho_2 - \Omega), \tag{30}$$

expressed in units of meters and days.

Under average conditions the coversand has a conductivity of 5 m/day, a thickness b′ of 10 m, and a wet perimeter B of the streams of first order and second order of about 10 m; eq. (23) then reduces to:

$$\Omega = 0.1 \text{ day/m} \tag{31}$$

Substitution of eq. (31) in (29) and (30) yields:

$$L_1 = 8Kb(586\rho_1 - 0.1 \text{ day/m}) \tag{32}$$
$$L_2 = 8Kb(293\rho_2 - 0.1 \text{ day/m}) \tag{33}$$

From these equations it is obvious that there is only a solution if (approximately):

$$\rho_1 > 1:6000 \text{ and } \rho_2 > 1:3000 \tag{34}$$

Physically it means that, if the relief factor ρ is smaller than these approximate values, the quotient of the hydraulic head Δh and the radial flow resistance LΩ becomes so small that in a real drainage situation the required groundwater discharge U will not be reached. The condition (see eq. (22)) is defined by:

$$\frac{\Delta H}{L\Omega} > U \tag{35}$$

Substitution of eq. (25) and (31) yields:

$$\frac{\rho}{U} > 0.2 \text{ day/m} \tag{36}$$

If this condition is not satisfied, diffuse discharge will occur. Diffuse discharge means a reduction of Ω through an increase in the wet perimeter of the discharge area.

The equations (32) and (33) yield: L_1 = 3860 m and L_2 = 1350 m for the average situation of the Pleistocene area for groundwater flow to the streams of first and second order with ρ_1 = 1 : 2000; ρ_2 = 1 : 1750; Kb = 2500 m^2/day and Ω = 0.1 day/m. These values agree with the observations. The surface slopes are above 1 : 1000 in the particular case of the ice-pushed ridges. If, moreover, the transmissivity amounts to a few thousand square meters per day, it can be concluded from eq. (29) and (30) that the stream spacing of the channels of first and second order would increase to such an extent that they would exceed the horizontal dimensions of the ice-pushed ridges. This constitutes the reason why at present a secondary drainage system is generally absent in the ice-pushed ridges.

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[3] In the absence of data on the average climatic situation during the Holocene, data on the present-day climate have been used in this study.

Streams of first and higher order may coincide on the other hand in cases of very slight relief and/or low permeability. Under extreme circumstances it gives rise to marshy conditions and the development of peat, which can be considered as a reduction of the stream spacing to zero. This situation is encountered on flat plateau areas.

Influence of a semi-impervious layer near the surface – On the plateaus of Drente and West Northbrabant semi-impervious clay deposits of the order of magnitude of the stream spacing L are present at the surface or at a shallow depth. Therefore the infiltrating precipitation has to overcome an extra vertical flow resistance Λ. In that case for the specific drainage resistance eq. (22) becomes:

$$\frac{\Delta h}{U} = \Upsilon = \frac{L^2}{8Kb} + L\Omega + \Lambda \tag{37}$$

where: $$\Lambda = \frac{b'}{K'_v} \tag{38}$$

in which b' represents the thickness of the semi-impervious deposits and K'_v its average vertical hydraulic conductivity.

Substitution of eq. (25) in (37) for U = N yields the following quadratic equation for L:

$$f(L) = \frac{\rho L}{2} - \left(\frac{L^2}{8Kb} + L\Omega\right)N = \Lambda N \tag{39}$$

If ρ is fixed, the following solution of eq. (39) is obtained:

$$\frac{L}{Kb} = \frac{2\rho}{N} - 4\Omega \pm \sqrt{\left(\frac{2\rho}{N} - 4\Omega\right)^2 - \frac{8\Lambda}{Kb}} \tag{40}$$

Eq. (39) possesses two roots for L with the same Λ-value. Since it is assumed that the drainage system developed from the larger streams to the smaller ones, the solution with the larger L will hold. For this reason it is not acceptable to have a minus sign before the radical sign. Moreover a minus sign leads to $L \to 0$ if $\Lambda \to 0$, which is contrary to what is expected.

There is only a solution of eq. (40) if:

$$\Lambda \leqslant \frac{Kb}{2}\left(\frac{\rho}{N} - 2\Omega\right)^2 \tag{41}$$

or: $$N\left(2\Omega + \sqrt{\frac{2\Lambda}{Kb}}\right) \leqslant \rho \tag{42}$$

Equation (39) reveals the meaning of this condition. It shows that with increasing Λ the equation can be satisfied through a decrease of L, but only as long as the reduction of the quadratic term on the left hand side of the equation remains larger than that of the first term. The maximum value of Λ according to eq. (41) is the critical value above which this is no longer the case:

$$\Lambda_{crit} = \frac{Kb}{2}\left(\frac{\rho}{N} - 2\Omega\right)^2 \tag{43}$$

The corresponding critical value of L is represented by its smallest value according to eq. (40). Hence:

$$L_{crit} = Kb\left(\frac{2\rho}{N} - 4\Omega\right) \tag{44}$$

Thus there exists no value which would satisfy eq. (39) for $\Lambda > \Lambda_{crit}$. Physically this means that if L becomes smaller than L_{crit}, the hydraulic head $\Delta h = \frac{1}{2}L\rho$, becomes smaller than the value which is necessary to overcome the vertical and horizontal groundwater flow resistances for the recharge amount N. In this case the rainfall surplus must be partly discharged above the covering layer by throughflow in the topsoil or by surface runoff.

The maximum vertical percolation rate can thus be determined for each Λ-value and given surface slope ρ. On the other hand eq. (42) sets a lower limit to the surface slope ρ, given the values for Λ and U. Eq. (44), when applied to the average data of the Pleistocene area with Kb = 2500 m^2/day, ρ = 1 : 2000 and Ω = 0.1 day/m, shows that in cases of slow percolation through a covering layer with a resistance of more than 500 days, even the annual mean precipitation surplus (0.85 mm/day) cannot percolate completely through this layer. This indicates that surface retention and discharge will occur frequently through the upper soil layers in most of the plateau areas. This explains the swampy conditions and the development of raised bogs on these plateaus.

Secondary drainage systems of third and higher order

The flow systems of higher order require less groundwater head to produce the same discharge since the resistance Υ decreases with decreasing stream spacing. Therefore the higher order systems will react more directly on rainfall of shorter duration and higher intensity than the systems of first and second order.

Since the increase of the n-day sum of the precipitation with n becomes progressively smaller, the required discharge capacity decreases as the number of days available for the discharge process increases. This number of days is determined by the storage capacity available at the beginning of a rainy period and by the discharge rate when the storage capacity is used up. In other words, the storage capacity and the prevailing discharge rate determine when the n-days sum reaches a critical value.

In order to avoid exceeding the groundwater storage capacity which would result in ponding and marshy conditions, the following condition has to be satisfied:

$$t\overline{U} = t\overline{i} - S \tag{45}$$

where: t = period length of rainfall;
$\overline{i}$ = average rainfall rate per time-unit;
$\overline{U}$ = average discharge;
S = storage.

The subsurface storage capacity depends on the groundwater depth. The total storage capacity between depth d and 0 is denoted as S(d). Wind (1967) proposed a method to find the maximum discharge required for the maximum amount of rainfall per day. A similar procedure is used here to obtain the required discharge for different frequencies of excess.

An equation of the type (see fig. 10):

$$i = c\left(\frac{t}{\Delta t}\right)^{-m} \qquad (46)$$

approximately expresses the relation between the mean amount of precipitation per time-unit that is exceeded with a given probability in some time interval, and the length of that time interval. Here i is the mean amount of precipitation per time-unit that is exceeded with a given probability p in a time interval t; c and m are local climatic parameters only depending on p; $\Delta t = 1$ day.

Substitution of eq. (45) in (46) yields:

$$\bar{U} = c\left(\frac{t}{\Delta t}\right)^{-m} - \frac{S}{\Delta t}\left(\frac{t}{\Delta t}\right)^{-1} \qquad (47)$$

The time t_{max} for which $\bar{U} = \bar{U}_{max}$ is obtained by taking:

$$\frac{d\bar{U}}{dt} = 0 \qquad (48)$$

If S is taken to be constant (= storage capacity available), this yields:

$$\bar{U}_{max} = (1\text{-}m)\, i\, (t)_{max} \qquad (49)$$

or $$\bar{U}_{max} = c(1\text{-}m)\left(\frac{S}{mc\Delta t}\right)^{\frac{-m}{1-m}} \qquad (50)$$

$\bar{U}_{max}$ is the average groundwater discharge that is required to avoid exceeding of the available groundwater storage capacity. For simplicity's sake we assume that the average discharge amounts to half its highest value during the rise of the groundwater table from the depth where discharge starts to the surface. In the following the spacing of the streams of different order will be considered as a function of the required discharge U_r at the highest groundwater level. Thus:

$$U_r = 2\bar{U}_{max} \qquad (51)$$

Storage capacity and required discharge – The storage capacity S(d) is defined as the volume of water that is stored per unit surface when the groundwater table rises from a given depth d to the surface. The storage factor μ is defined as the volume of water that an aquifer releases or takes into storage per unit surface area of an aquifer per unit change of depth of the groundwater table. In formula:

$$S(d) = {}_0\!\int^d \mu d\,(\text{depth}) \qquad (52)$$

The storage factor μ is a function of: the composition of the soil, the successive cycles of wetting and drying, and the depth of the groundwater table. Therefore its value varies with time and location and is very difficult to estimate.

The groundwater depth (d) versus storage (μ) relations of different investigations were used to formulate the following approximation (see d e V r i e s, 1974):

$$\mu = ad \qquad (53)$$

with $a = 0.1\ m^{-1}$. Because the maximum value of μ is approximately 0.2, eq. (53) is limited to $d \leqslant 2$ m.

The storage capacity S(d) with $0 < d < 2$ m is found by integration:

$$S(d) = \tfrac{1}{2}ad^2 = 0.05\ d^2 \quad (\text{d and S in meters}) \qquad (54)$$

A combination of eqs. (50) (51) and (54) yields the required discharge U_r as a function of the groundwater depth d:

$$U_r = 2c(1\text{-}m)\left(\frac{0.05d^2}{mc\Delta t}\right)^{\frac{-m}{1-m}} \qquad (55)$$

In The Netherlands the streams of higher order are predominantly responsible for discharge during the winter period, when the evaporation is negligible. The order of the streams which start to participate in the groundwater discharge process will increase with a rise of the groundwater table and thus with an increasing rainfall rate (fig. 2). The rainfall rate is characterized by its frequency of occurrence, and thus by the parameters m and c in eq. (55).

At locations of groundwater outcrops streams of different order developed. During this process the available groundwater storage capacity S was determined for any stream of n^{th}-order by its incision depth d'_n. This applied as long as the $(n+1)^{th}$-order streams were not developed. Thus the storage capacity S, described by eq. (54), is:

$$S = S(d'_n) = 0.05\ {d'_n}^2 \qquad (56)$$

It is assumed that during a rainy period the rise of the water level in the stream channels of third and higher order is negligible with respect to the rise of the groundwater table. In this case the maximum possible groundwater head at the

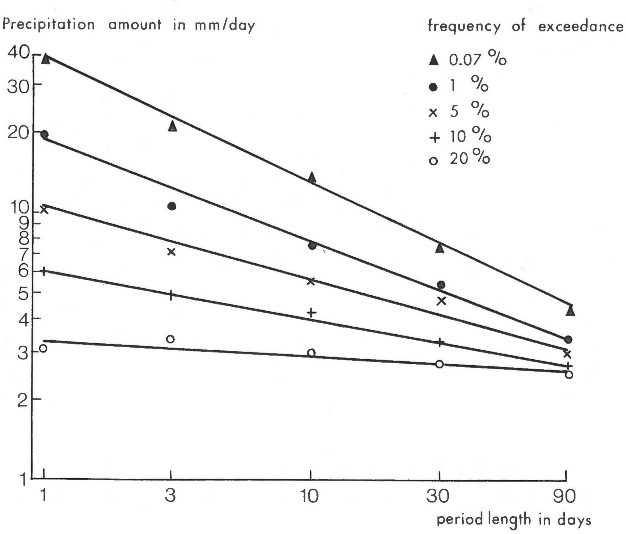


Fig. 10
Frequency of excess of the average winter-period precipitation amount per day versus period length for Winterswijk over the period 1880-1953.

divide (i.e. half-way between the drainage channels) minus the head in the streams of a given drainage system, equals the groundwater depth d_n at the divide and at the moment when the streams start to participate in the groundwater drainage process. And d_n is approximately equal to the incision depth of the streams of the groundwater flow system. This applies as long as the $(n+1)^{th}$-order streams have not developed. In this case Δh in eq. (3) can be replaced by d'_n and thus the drainage resistance Υ_n can be described by the relation:

$$\Upsilon_n = \frac{d'_n}{U_{rn}} = \frac{d_n}{U_{rn}} \qquad (57)$$

Henceforth d_n is called the initial groundwater depth of the drainage system of n^{th}-order.

The unsaturated zone above the groundwater table is near saturation during periods of high discharge when the groundwater table is near the surface. It is assumed that under those circumstances, the total amount of water stored per unit area as a result of a rise of the groundwater table is the same from the divide to the drainage channel.

Substituting eqs. (56) and (55) in eq. (57) gives the maximum drainage resistance which corresponds to the required drainage during the development of the streams of different orders:

$$\Upsilon_n = \frac{d_n \left(\frac{0.05\, d_n^2}{mc\,\Delta t}\right)^{\frac{m}{1-m}}}{2c - 2mc} \qquad (58)$$

Stream spacing of third and higher order systems – Eq. (37) gives the relation between stream spacing (or drainage density) and drainage resistance:

$$\Upsilon = \frac{L^2}{8Kb} + L\Omega + \Lambda \qquad \text{(see 37)}$$

Solving L from eq. (37) yields:

$$L = -4Kb\Omega + \sqrt{(4Kb\Omega)^2 + 8Kb(\Upsilon - \Lambda)} \qquad (59)$$

Substitution of eq. (58) in (59) provides a means of determining the stream spacing L. This stream spacing is defined as the spacing at which precipitation with a given probability can be discharged as groundwater. This results in the following relations for the stream spacing of the drainage systems of third and higher order:

$$L_n = 2\sqrt{(2Kb\Omega)^2 - 2Kb\Lambda + \frac{Kbd_n}{c-mc}\left(\frac{0.05d_n^2}{mc\Delta t}\right)^{\frac{m}{1-m}}} - 4Kb\Omega \qquad (60)$$

In analogy with eq. (41) the precipitation can only percolate through the confining layer if the first term in eq. (60) is real. This condition is satisfied if:

$$\Lambda < 2Kb\Omega^2 + \frac{d_n}{2(c\text{-}mc)}\left(\frac{0.05d_n^2}{mc\Delta t}\right)^{\frac{m}{1-m}} \qquad (61)$$

An approximate correlation is obtained between stream order and precipitation probability from an empirical overall knowledge of the hydrological situation in The Netherlands. With the aid of Horton's law (H o r t o n, 1945), a subsequent refinement has been made by a procedure of iteration. This law gives within one stream network the relation between stream spacings of different order. The precipitation probability is characterized by the parameters m and c, which are available for The Netherlands (fig. 10, eq. 46). The factor d_n is obtained from a very large amount of data on groundwater depths, available in The Netherlands. For applications reference should be made to d e V r i e s (1974).

Drainage resistance of systems with a small spacing – In general the total thickness of the aquifer is not used for groundwater flow if streams are located less than about 500 m from each other. In such a case only part of the aquifer participates in the transmission of the discharge. The size of this part depends on both the stream spacing and the hydraulic properties of the subsoil. According to H o o g h o u d t (1940), the maximum depth b^+ to which a homogeneous and isotropic aquifer contributes to the discharge is given by the relation:

$$b^+ = \tfrac{1}{2}\, L^* \qquad (62)$$

Eq. (63) shows the relation between the penetration depths Z_1 and Z_2 of streamlines in an isotropic aquifer and in an anisotropic aquifer with identical geometrical properties:

$$Z_1 : Z_2 = 1 : \sqrt{\frac{K_v}{K_h}} \qquad (63)$$

where K_v and K_h are the vertical and horizontal hydraulic conductivities of the anisotropic aquifer. Substitution of eq. (62) in (63) gives the depth b^+ for a homogeneous and anisotropic aquifer:

$$b^+ = 0.25\, L\sqrt{\frac{K'_v}{K'}}, \qquad (64)$$

where K'_v and K' are the vertical and horizontal hydraulic conductivities of the upper part of the aquifer, including the coversand layer.

Similar to eq. (22):

$$\Upsilon = \frac{L^2}{8K'b^+} + L\Omega \qquad (65)$$

where K' is the horizontal hydraulic conductivity of the upper part of the aquifer. For the coversand the ratio $K'_v/K \approx 0.5$. Substitution of this ratio in eq. (64) yields:

$$b^+ = 0.18\, L, \qquad (66)$$

$$\text{hence } \Upsilon = \frac{L^2}{1.4K'L} + L\Omega = 0.7\frac{L}{K'} + L\Omega \qquad (67)$$

Supposing that the thickness of the covering layer $b' = 10$ m, and that the wet perimeter of the stream channel $B = 2$ m, then eq. (23) yields:

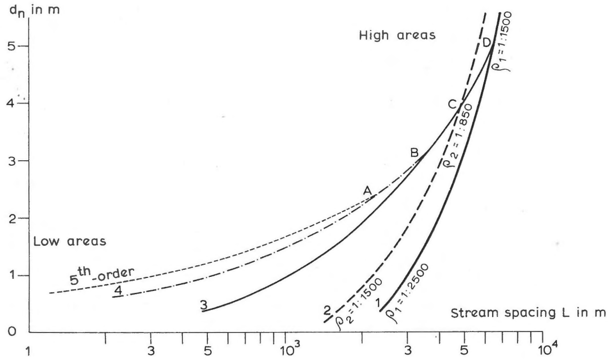


Fig. 11
Stream spacing of different order depending on initial groundwater depth d_n and the average surface slope of first and second order ρ_1 and ρ_2. With increasing groundwater depth d_n, stream systems of higher order are not developed or vanish. A, B and C represents the limiting groundwater depths at which the calculated stream spacings of n^{th}-order equals the stream spacings of $(n-1)^{th}$-order.

$$\Omega = \frac{3.2}{\pi K'} \tag{68}$$

and consequently: $\Upsilon = \dfrac{1.7L}{K'}$ (69)

A combination of eq. (69) with (58) yields:

$$L_n = \frac{K' d_n \left(\frac{0.05 d_n^2}{mc\Delta t}\right)^{\frac{m}{1-m}}}{3.4c(1-m)} \tag{70}$$

The stream spacings of different order have been determined through the equations (32), (33), (60) and (70) from the basic physical landscape properties for various landscapes in the Pleistocene area. The relations between stream order and the appropriate rainfall frequencies (parameters m and c), have been used as a tentative working hypothesis (de Vries, 1974), and are shown in table 2.

Fig. 11 shows the relation between groundwater depth d_n and stream spacing for streams of different order. The graphs are based on the data of table 2 and Kb = 2500 m^2/day, K' = 5 m/day, Ω = 0.1 day/m and Λ = 0. This figure shows that with increasing groundwater depth, the stream systems coincide. The required discharge of a drainage system of third and higher order is apparently reduced with increasing groundwater depth because of the increase in number of days available for discharge of the rainfall surplus as groundwater. With this increase, the amounts of rainfall per time-unit approach each other according to eq. (46) for different frequencies (fig. 10). Thus it may be expected that the required discharge of a drainage system comes in an area with a relatively deep groundwater table, close to that which should be attributed to a system of the next lower order. Consequently, stream systems of higher order vanish or are not developed with increasing average groundwater depth. This

Table 2.
Rainfall frequency characteristics and the associated stream orders.

stream order	frequency of excess winter period	c	m	equation used for fig. 11
1	mean annual precipitation	0.85 mm/day	--.--	(32)
2	50%	1.7 mm/day	0.00	(33)
3	15%	4.5 mm/day	0.12	(60)
4	5%	10 mm/day	0.25	(70)
5	1%	20 mm/day	0.38	(70)

Table 3.
Stream spacing L, drainage resistance Υ, initial groundwater depth d and areal flux $\bar{U}$. Assumptions: Kb = 2500 m^2/day; K' = 5 m/day; Ω = 0.1 day/m; ρ_o = 1 : 2500; ρ_1 = 1 : 2000; ρ_2 = 1 : 1750.

Primary drainage system		L = 2L*	Υ	Δh or d	$\bar{U}$
	Fluvial/eolian topography	60 km	3 x 10^5 days	Δh = 17 m	0.05 mm/day
	Ice-pushed topography	30 km	3 x 10^4 days	Δh = 40 m	1.00 mm/day
	Zero order streams	10 km	5 x 10^3 days	Δh = 2 m	0.40 mm/day
Secondary drainage system					
	First order streams	4000 m	1200 days	Δh = 1.00 m	0.85 mm/day
	Second order streams	1350 m	230 days	Δh = 0.40 m	1.75 mm/day
	Third order streams	500 m	60 days	d = 0.40 m	6.70 mm/day
	Fourth order streams	100 m	35 days	d = 0.30 m	8.6 mm/day
	Fifth order streams	25 m	8 days	d = 0.25 m	30.0 mm/day

trend may continue if the depth of the groundwater table continues to increase, until the required discharge is reduced to the annual average precipitation surplus. This condition represents the end-member of a range of possible cases. It is encountered in the ice-pushed ridges with a deep groundwater table. In these areas the discharge of springs at the edges remains constant at 1 mm/day throughout the year. The characteristics of the different drainage systems are compiled in table 3.

The stream network

Bifurcation ratio – Horton (1945) showed that in any stream system the ratio between the number of streams of any order and the number of streams of the next higher order is approximately constant. This ratio is called the bifurcation ratio. Assuming a regular distribution of streams in a basin, the number of streams is inversely proportional to stream spacing L, thus:

$$\beta^m = \frac{L_{n-m}}{L_n} \quad (71)$$

Substitution of the values of L_n from eqs. (29), (30), (60) and (70) makes it possible to determine the bifurcation ratio from the basic physical landscape properties. For example: the third order drainage system seems to have been adapted to a rainfall frequency of about 15%. The "intensity duration" relation (eq. 46) of this precipitation probability is characterized by m = 0.125 and c = 4.5 mm/day. Substitution of these values in eq. (60) with $\Lambda = 0$ yields:

$$L_3 = -4Kb\Omega + \sqrt{(4Kb\Omega)^2 + 1760\, d_3{}^{1.24}} \quad (72)$$

A combination of eqs. (29), (71) and (72) gives:

$$\beta^2 = \frac{8Kb\,(586\rho_1 - \Omega}{-4Kb\Omega + \sqrt{(4Kb\Omega)^2 + 1760\, Kbd_3{}^{1.24}}} \quad (73)$$

The following results are obtained for average conditions of the Pleistocene area, with Kb ranging from 500 – 5000 m²/day, ρ_1 = 1 : 2000, Ω = 0.1 and d_3 ranging from 0.25 m to 0.75 m:

	d_3	
Kb	*0.25*	*0.75*
500	$\beta = 2$	$\beta = 1$
2000	$\beta = 3$	$\beta = 2$
5000	$\beta = 5$	$\beta = 2$

Thus the theoretical model shows an increase in the bifurcation ratio with an increase of transmissivity and a decrease of groundwater depth.

Stream net density – Stream net density of a basin is defined as the total length of streams per unit area of the basin. Thus the stream net density D_n of a stream system of the nth-order can be defined by the spacing of the streams of nth-order,

$$D_n = \frac{1}{L_n} \quad (74)$$

In reality, however, the catchment is not uniformly drained by streams of different order. Contrary to what one would expect it has often been observed that streams of higher order do not develop in a part of the catchment near a stream of lower order. In addition, each catchment has its divides which are generally devoid of channels. This occurs especially if the divide is part of a relatively elevated topographic feature. This is exemplified by the broad coversand ridges in the Pleistocene area. This implies that the sum of the areas of basins of nth-order is less than the total area of the basin of (n –1)th-order, so that the overall density of streams of a given order is less than that calculated on the basis of the stream spacing. Leopold, Wolman & Miller (1964, p. 147) give examples of a reduction of stream density by 50 to 70%. Therefore we write instead of eq. (74):

$$D_n = \frac{p^n}{L_n} \quad (75)$$

where p is a reduction factor (p < 1).

A more general expression may be derived with D_n as the function of the spacing of any stream order using the bifurcation ratio. Substitution of eq. (71) in (75) yields:

$$D_n = \frac{p^n \beta^{n-m}}{L_m} \quad (76)$$

Equation (76) provides a general means of considering the interrelationship of D_n and the different combinations of basic hydrological properties which control the stream spacings of different orders. In the Pleistocene area of The Netherlands those sub-areas of low stream density are found especially near the divides of first order, e.g. coversand ridges. The streams of higher order are well developed in the depressions between ridges. They show a rather uniform distribution. Therefore the reduction factor p should only be applied to compute the densities of streams of first and second order from zero and first order. Thus for the conditions prevailing in The Netherlands eq. (76) has to be changed to:

$$D_n = \frac{p^2 \beta^{n-m}}{L_m} \quad (n \geqslant 2) \quad (77)$$

The reduction factor p has been found empirically by substitution of observed data. The following average values have been obtained from 1 : 50.000 maps of the Pleistocene area: $L_1 \approx 4000$ m, $D_3 \approx 1.3$ km^{-1} and $\beta \approx 4$. Substitution of these data in eq. (77) yields p = 0.57.

Comparison of the actual stream network with the theoretical model; discussion

The actual stream network on the Pleistocene landscape has been analysed for bifurcation ratio and stream density. The results are compiled in fig. 12, where the bifurcation ratio β_{14} is plotted versus the stream density. This leads to the following conclusions:

– The Drente Plateau and the West Northbrabant Plateau (west) exhibit the lowest stream density. This is most probably due to the presence of resistant plateau-areas of loamy material which constitute broad divides which the headward erosion has not yet dissected. The development of raised bogs on these flat divides points to a lack of sufficient secondary relief and to a low permeability. Fig. 13 shows the inadequate drainage conditions with many ponds on the West Northbrabant Plateau a hundred years ago.

– The stream density in the Kempen area is also relatively low. This can be explained by the relatively high permeability of the subsoil because the coversand blanket is thin and the coarse deposits of the Pleistocene fluvial fan are close to the surface.

– The highest stream density is associated with marshy areas and shallow groundwater tables. At the edges of the Peel area where diffuse upward seepage from the Peel divide may occur, high stream densities have been observed as well. Low values of the drainage density are found on those marshy areas which have not been dissected by streams, because of stagnant water and peat growth.

– In general, the bifurcation ratio seems to decrease when the stream density increases. This tendency, which does not agree with the theoretical expectations (eq. 77), is due to the

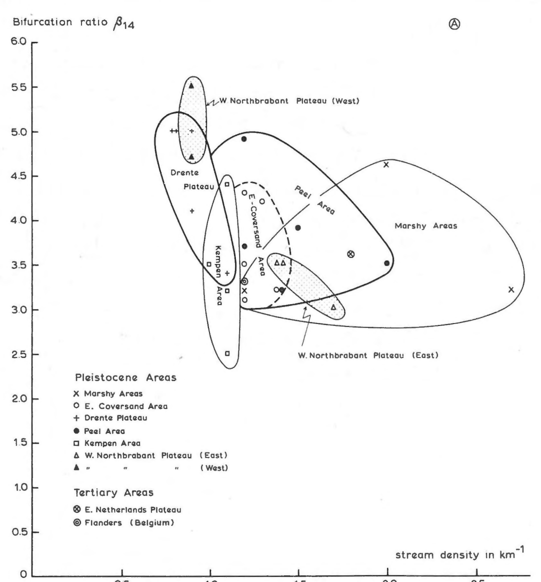


Fig. 12
Relation between the bifurcation ratio β and the stream density D, obtained from the stream network on the Waterstaat Map 1:50 000.

Fig. 13
Example of the inadequate drainage conditions on the West Northbrabant Plateau; from the topographical map 1 : 50 000, issued in 1850, sheet Breda.

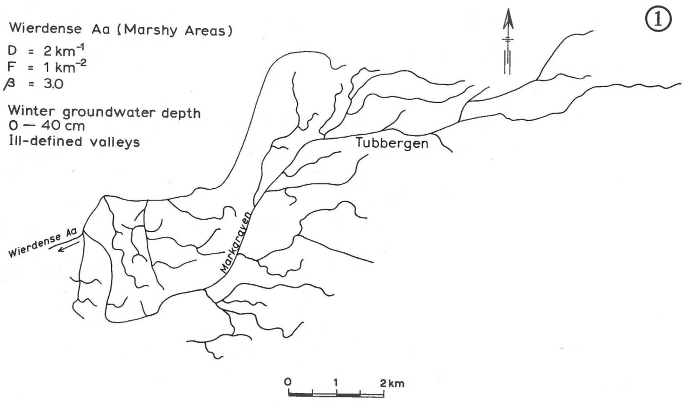
occurrence of the highest bifurcation ratio in the areas with the lowest stream density (Drente Plateau and West Northbrabant Plateau). The stream density in these areas is considerably lower than it "should be". The high bifurcation ratio can be explained by the effect of a concentration of stream branches in the main valleys. This gives a feather-shaped stream network. The stream nets in other areas show better developed dendritic structures (fig. 14).

To compare the observed stream-network characteristics with the results computed from the theoretical model, table 4 has been compiled. The bifurcation ratio appeared not to be constant throughout the stream network, mainly because of the topographic inhomogeneity within one drainage basin. This is why β has been determined separately for different parts of a stream net. β_{mn} means the bifurcation ratio that has been determined by considering the streams of n^{th} – and m^{th}-order, so that:

$$\beta_{mn} = \sqrt[m-n]{\frac{L_n}{L_m}}$$

In table 4, D^*_3 represents the stream density according to eq. (77) with β calculated as β_{34}; D_3 has been calculated with the aid of eq. (75).

Table 4 shows that on average there is a reasonable agreement between the calculated and observed stream-net charac-



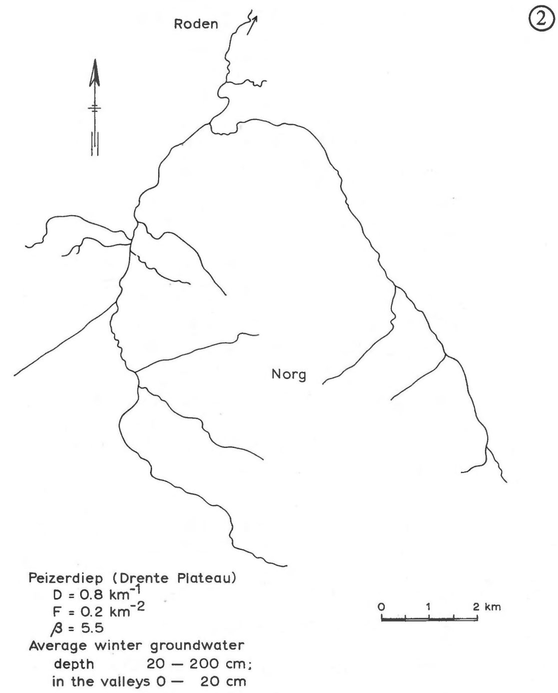


Fig. 14
Examples of stream nets, obtained from the Waterstaat Map 1 : 50 000. 1 represents a dense and dendritic system; 2 represents a wide and feather-shaped system. F = stream frequency, it represents the number of stream segments per unit area.

teristics for the Pleistocene area as a whole. The average observed bifurcation ratio amounts to 4.1 whereas the average calculated bifurcation ratio reaches 4.0. The observed and calculated figures for the stream density are 1.3 and 1.7 km^{-1} respectively.

Notable discrepancies occur between the calculated and actual stream-net properties of the individual sub-areas. The non-equilibrium conditions have already been explained for the loamy plateaus and for the marshy areas. The effect of the irregular, non-fluvial coversand topography could not be sufficiently included in the model. The undulation of the aeolian topography is reflected in the form of the groundwater table at shallow depth. This leads to an irregular distribution of the storage capacity. The groundwater table more or less follows the topography. It creates a kind of sub-system of areas of groundwater discharge and areas of groundwater recharge. This groundwater flow sub-system and the groundwater flow systems towards the drainage channels are superimposed (Ernst, de Ridder & de Vries, 1970).

A further refinement of the method can be obtained by applying a network calculation that includes the minor topographical elements. An additional investigation should be carried out to find a more appropriate relation between stream order and rainfall frequency.

Table 4.
Average values of the bifurcation ratio and the stream density. For the basic physical data of the different landscapes on which the calculations are based, the reader is referred to de Vries, 1974.

	Bifurcation ratio				Streamnet density (km^{-1})		
	calculated		observed		calculated		observed
	β_{13}	β_{34}	β_{14}	β_{34}	D_3	D_3*	$D_3(D_4)$
Marshy Areas Kb = 1500		5.7	4.2	3.7	2.7	2.7	2.0
Kb = 3000		6.2			1.4	1.4	
Eastern Coversand Area	1.9	6.6	3.2	3.7	0.6	7.5	1.3
Drente Plateau	3.2	5.9	4.9	4.4	0.5	1.6	0.9
Peel Area	2.7	3.1	3.8	3.8	0.3	0.4	1.5
Kempen Area	2.7	4.1	5.1	3.4	0.4	1.0	1.1
W-Northbrabant Plateau:							
West			4.4	5.1			0.9
	3.0	5.4			0.8	2.4	
East			3.4	3.3			1.5
Average of six landscapes	2.7	5.2	4.2	3.9	0.8	2.5	1.3

APPENDIX: LIST OF NOTATIONS

Symbol		dimension
B	Wet perimeter of a stream channel, or channel width in case of a wide and shallow stream	L
b	Thickness of an aquifer	L
b^+	That part of an aquifer which contributes to groundwater discharge of a given groundwater flow system	L
b'	Thickness of a less-pervious covering layer	L
c	Local constant of the rainfall amount-duration relation	$L.T^{-1}$
d	Depth of the groundwater table below the surface	L
d'_n	Incision depth of a stream channel of n^{th}-order below the surface	L
d_n	Groundwater depth from which the drainage system of order n starts to participate in groundwater discharge (initial groundwater depth); $d_n \approx d'_n$	L
h	Hydraulic head of groundwater	L
Δh	Difference in hydraulic head between a divide and a discharge base	L
i	Rainfall amount per time unit	$L.T^{-1}$
K	Permeability coefficient (hydraulic conductivity) of an aquifer	$L.T^{-1}$
K'	Permeability coefficient of a less-pervious covering layer	$L.T^{-1}$
L^*	Horizontal distance between the divide and the discharge base of a groundwater flow system	L
L_n	Spacing between a parallel set of streams of order n	L
m	Local constant of the rainfall amount-duration relation	dimensionless
N	Precipitation surplus per unit area	$L.T^{-1}$
q	Flux per unit width	$L^2.T^{-1}$
q_o	Flux per unit width at the point of discharge	$L^2.T^{-1}$
s^*	Surface slope	dimensionless
s	Slope of the groundwater table	dimensionless
S	Change of groundwater storage per unit area through a change of the phreatic level	L
S(d)	Groundwater storage capacity as function of the groundwater depth d	L
U	Mean areal groundwater discharge, or average flux through the phreatic surface ($U = q_o/L^*$)	$L.T^{-1}$
β	Bifurcation ratio	dimensionless
Λ	Vertical flow resistance of a less-pervious layer ($\Lambda = b'/K'_v$)	T
μ	Storage coefficient	dimensionless
ρ	Mean surface slope between the divide and the valley bottom of a parallel stream system	dimensionless
Υ	Specific drainage resistance	T
Ω	Radial resistance of groundwater flow towards a channel	$T.L^{-1}$

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