# ESTIMATION OF THE TECTONIC STRAIN RATIO FROM THE MEAN SHAPE OF DEFORMED ELLIPTICAL MARKERS

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## ABSTRACT

Lisle, R.J. (1977). Estimation of the tectonic strain ratio from the mean shape of deformed elliptical markers. Geol. Mijnbouw, 56, p. 140-144.

By considering computer models representing suites of passive elliptical markers subjected to homogeneous deformation, the relationship between the strain ellipse shape (Rs = $1+e_1/1+e_2$) and mean final axial-ratio of the markers is investigated. Random and uniform models in terms of the choice of pre-tectonic initial axial-ratios (Ri) and marker-orientation are considered. The arithmetic mean ($\bar{R}$), geometric mean (G) and harmonic mean (H) of the final axial-ratios (Rf = long/short axis) are calculated and it is found that $\bar{R}$ departs the most from Rs. The closest of the means to the value of Rs is given by H. Using H as an estimate of Rs always gives rise to an error when the markers had a non-circular original shape. However this error diminishes relatively with increase in Rs and decrease in Ri.

For the initial clast shapes present in coarse-grained detrital sedimentary rocks and Rs equal or greater than 2.5, H allows an estimation of Rs within 10% error.


## INTRODUCTION

For a suite of circular passive markers subjected to homogeneous deformation the final shape of each marker is a direct indication of the shape of the strain ellipse (Rs = $1+e_1/1+e_2$). The mean axial ratio for the suite is the best estimate of Rs.

However, circular markers are rare in geology with sedimentary clastic grains, pebbles and even oöids usually possessing initial eccentricities. Special methods of strain analysis devised for suites of markers with an initial elliptical shape include those of Ramsay (1967, p. 202-211), Dunnet (1969), Elliott (1970), Dunnet & Siddans (1971), Mathews *et al.* (1974) and Lisle (1977). These methods use both shape and orientation data from the deformed markers and all involve time-consuming techniques.

This paper describes the results of a study of the relationship between the mean of the final axial ratio of the deformed marker (Rf) and the axial ratio of the strain ellipse (Rs). The study attempts to find the answer to two questions:

Firstly, time (or computer-time) can be saved with some of the strain analysis techniques above if a rough estimate of Rs is already known e.g. the "onion" curve method of Dunnet (1969) and the Θ-curve method of Lisle (1977). Is it valid to use the mean Rf as an estimate of the tectonic strain in rocks where the markers had a pre-tectonic shape?

Secondly, the methods applicable to initially circular markers are much quicker and require less information than those for initially elliptical markers and it is tempting to make the assumption of "no initial distortion" in order to speed up the calculations. What magnitude of errors do we risk making if we use this assumption for rocks where an initial shape factor is involved?

We will now look at the various means of Rf which have been used for the calculation of Rs.

### *The Arithmetic Mean, ($\bar{R}$)*

$$\bar{R} = \frac{\Sigma Rf}{n}$$

This is likely to be a satisfactory way of estimating Rs for markers which were originally circular in section and was used by Cloos (1947) in his famous study of oölite deformation. Hossack (1968) calculated arithmetic means of pebble axial ratios in deformed conglomerates but admitted that ignoring the original shape factor probably causes a large error in the strain determination. Nevertheless the literature

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shows that $\bar{R}$ is still frequently used as a measure for assessment of the deformation.

A similar quantity to $\bar{R}$ is given by the slope of the line of best fit, passing through the origin, for the points on a scatter diagram of long against short axis length. This, for example, was calculated to find the strain in deformed agglomerates by Hobson (1971), and with deformed concretions and pebbles by Sanderson (1972). The relationship of this parameter to $\bar{R}$ is discussed by Mukhopadhyay (1973). Ramsay (1967, p.214) questioned the validity of the arithmetic mean of axial ratios as an indicator of the strain ellipse shape where initial eccentricity of the markers is involved.

*The Geometric Mean, (G)*

$$G = \sqrt[n]{Rf_1 \times Rf_2 \times Rf_3 \times \ldots\ldots Rf_n}$$

Dunnet (1969) pointed out that strain evaluation with the arithmetic mean of Rf is invalid and considered the geometric mean to be a crude approximation to, and an overestimation of, Rs. Barr (1976) used the geometric mean of pebble axial ratios in a deformed conglomerate within the Zambezi Belt.

Helm & Siddans (1971) calculated $\bar{R}$, G and Rs (using Dunnet's method for elliptical markers) for six specimens of deformed lapillar tuff. The $\bar{R}$ values showed a good deal of scatter compared with Rs. The G results were intermediate between those based on $\bar{R}$ and Rs.

*The Harmonic Mean, (H)*

$$H = \frac{n}{\Sigma \frac{1}{Rf}}$$

The harmonic mean of Rf has not previously been used as a measure of the strain. It has always the following relationship to the other means:

$$\bar{R} \geqslant G \geqslant H$$

## CALCULATIONS

In order to investigate the relationship between the mean Rf and Rs, the strain ratio, two mathematical models were considered. Each consisted of suites of variably-oriented elliptical markers of axial ratio Ri deformed by a homogeneous pure shear strain to give markers of axial ratio Rf.

*The Uniform Model*

The model consisted of 89 elliptical markers with the same initial axial ratio and with a uniform orientation distribution so that the 1st, 2nd, 3rd...89th grains were oriented initially at an angle of 1°, 2°, 3°...89° respectively to the principal direction of strain.

A computer programme based on the equations for deformation of an ellipse given by Ramsay (1967, p. 91-93) calculated the effect of strain on each marker in terms of the final shape Rf. Then, for the whole suite the means ($\bar{R}$, G and H) were calculated. This was repeated several times with changed values of Ri and Rs so that the dependence of $\bar{R}$, G and H values on Ri and Rs could be found. The results are shown in Fig. 1.

*The Random Model*

With this model, the pre-deformation system consisted of 50 markers, whose individual Ri values were chosen randomly within the range 1.1-2.5. Each individual marker orientation was chosen randomly within the range 0°-90° from the principal strain direction. The resulting set of undeformed markers had an arithmetic mean of Ri = 1.73 and Ri had a standard deviation of 0.45.

By means of the computer programme, this suite of markers was deformed by various strains (Rs) in turn, and the means of Rf ($\bar{R}$, G and H) calculated. The results are shown in Fig. 2.

## RESULTS

The results of both models described above can be summarized in the following way:

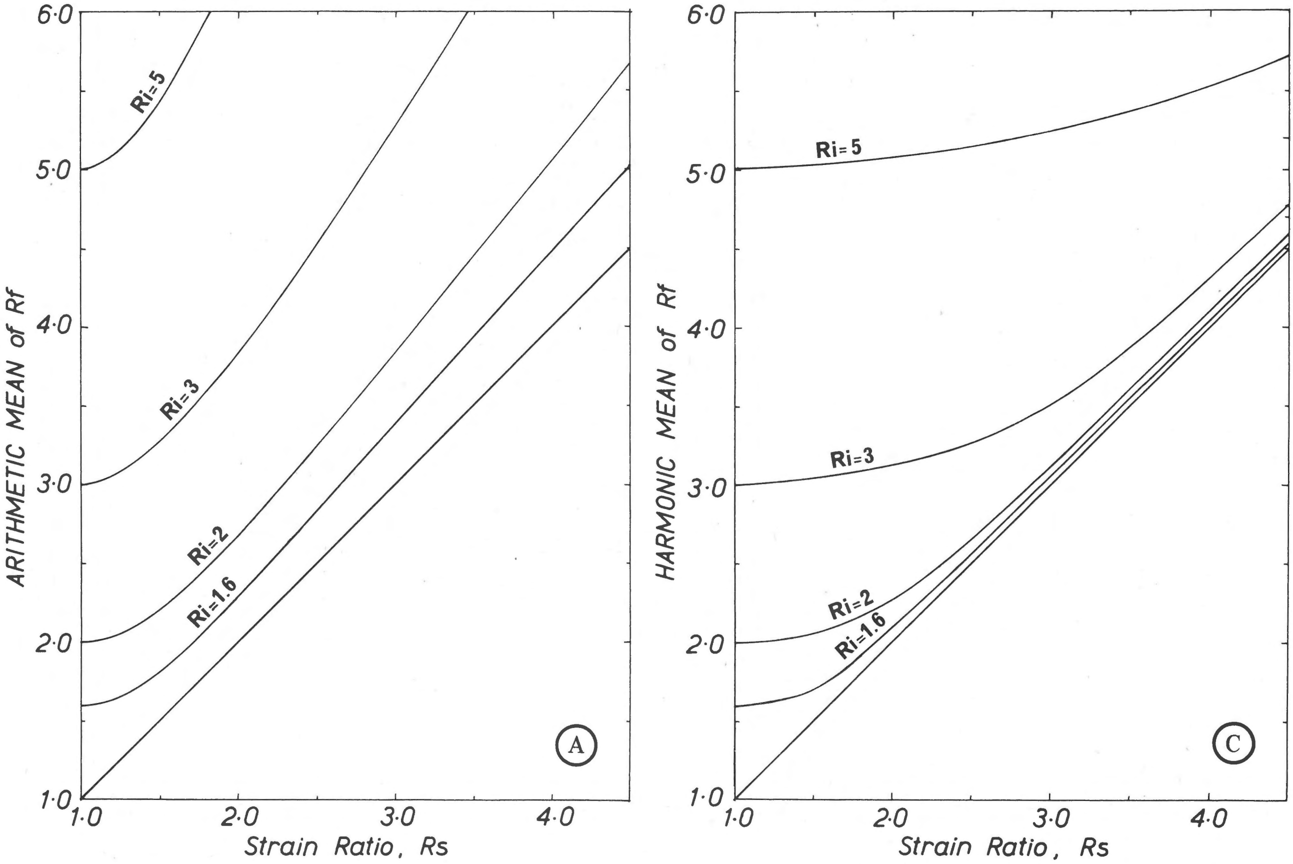
1. When an initial shape factor is involved (i.e. $Ri \neq 1.0$) no simple mean of Rf values gives directly the strain ellipse shape, Rs.
2. However of the three means, H was the closest to Rs, followed by G. $\bar{R}$ departed the most from Rs.
3. In general, each mean has a higher value than Rs. An exception occurred with the random model at high strains (Rs = 10.0) where H took a value less thans Rs (9.98).
4. Using any of the means of Rf as an estimate of Rs will, in general, result in an error. This error can be expressed as a percentage of the value of Rs.

   $$\%\ \text{error} = \frac{|\text{mean} - \text{Rs}|}{\text{Rs}} \times 100$$

   This % error decreases as Rs increases. This is the most pronounced with the harmonic mean (Fig. 3).
5. The % error decreases also as Ri decreases, i.e. the lower the initial shape factor the closer the mean approaches the strain ratio, Rs.

## APPLICATION TO A DEFORMED CONGLOMERATE

Data were collected from a deformed conglomerate on Maskostjakke, Norra Storfjället, Västerbotten, Sweden. These rocks, which probably belong to the Seve-Köli Nappe complex, are currently being mapped by students from the University of Leiden. The pebbles, mainly consisting of



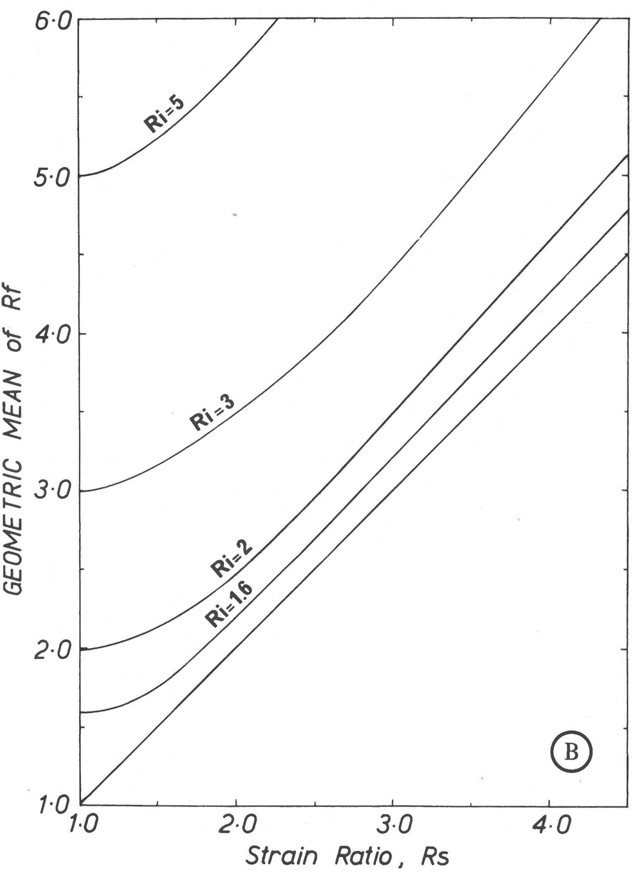




Fig. 1
The Uniform Model- the relationship between mean axial ratio of a suite of deformed elliptical markers and the tectonic strain ratio. The markers all had the same initial axial ratio (Ri) and a uniform pre-deformation orientation distribution of their long axes (see text). A, Arithmetic mean, B, Geometric mean. C, Harmonic mean.

quartzite in a quartz-rich matrix, show a distinct extension in a horizontal NW-SE direction. 400 two-dimensional pebble axial ratios and orientations were measured on a joint face approximately perpendicular to the long axes of the pebbles. A technique similar to the Rf/∅ method of Dunnet (1969) with modifications involving the use of Θ-curves (Lisle, 1977) was used to estimate the two-dimensional strain ratio. The Rf/∅ results gave a best-fit Rs value of 2.40 and allowed the estimation of initial pebble shapes (c.f. Dunnet 1969). These calculated initial shapes are shown in a frequency diagram (Fig. 4).

Table 1 compares the mean Rf values for the pebbles with the best-fit Rs value. The harmonic mean (2.45) was the closest of the means to the Rs value calculated by the Rf/∅ technique. Although 400 axial ratios were measured here for application of the Rf/∅ technique which is used as a comparison, there is no reason to believe that this large number of readings is required to obtain consistent values for the harmonic mean.

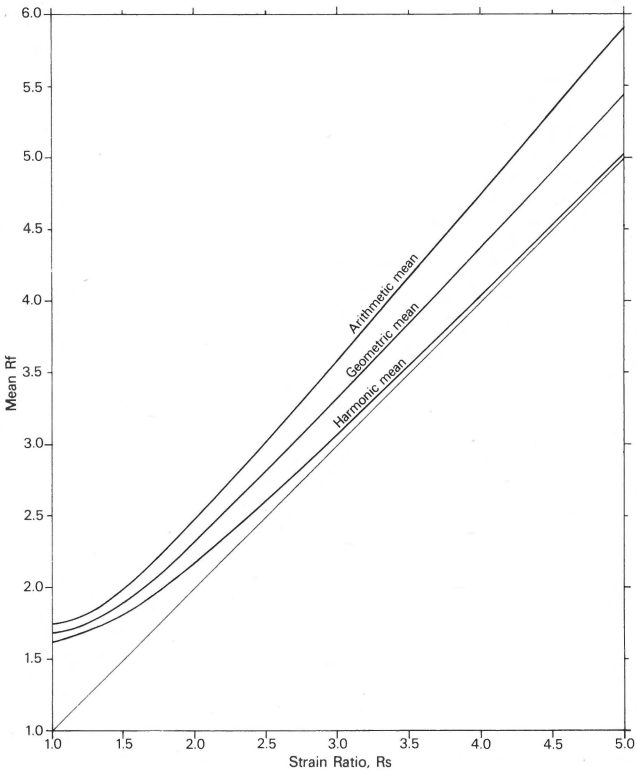



Fig. 2
The Random Model- the relationship between mean axial ratio of a suite of deformed elliptical markers and the tectonic strain ratio. Pre-deformation shapes and orientations of the markers were randomly chosen (see text).

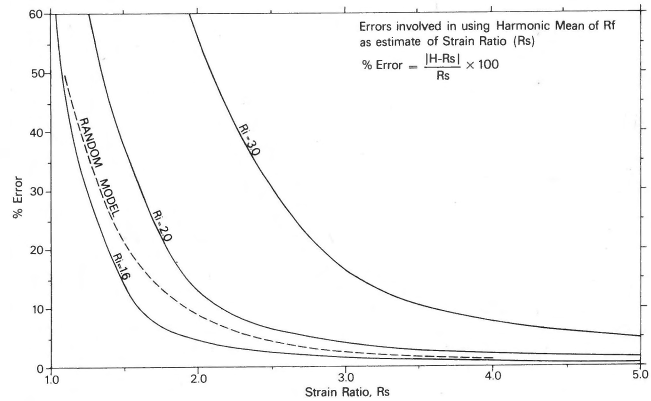



Fig. 3
Departure of the harmonic mean (H) of the final ellipse shapes from the strain ratio as predicted by the uniform (solid lines) and random model (dashed line).

## CONCLUSIONS

No simple mean of the final shapes of a homogeneously-deformed suite of elliptical markers will give the strain ellipse shape Rs. In particular, for high initial shape axial ratios and low strain ratios the use of the mean of Rf will yield wildly inaccurate results.

However, for the moderate pre-tectonic axial ratios shown by many sedimentary particles (Fig. 4 and, for example, Griffiths 1967, p. 120-125) good estimates of the strain ratio can be obtained from the harmonic mean of the post-tectonic axial ratios providing that certain assumptions are valid. These assumptions require that the strain be homogeneous, that the long axes lack a pre-deformational preferred orientation and that Rs is reasonably high (Fig. 3). The assumption of no initial preferred orientation imposes an important limitation on strain methods based on mean particle shape. It would be worthwhile now to investigate the magnitude of the errors produced by preferred orientations of the intensity commonly found in clastic sedimentary rocks.

Because the strain ratio determined from the mean Rf is by nature, an approximation, this method of strain analysis is obviously inferior to methods which take direct account of

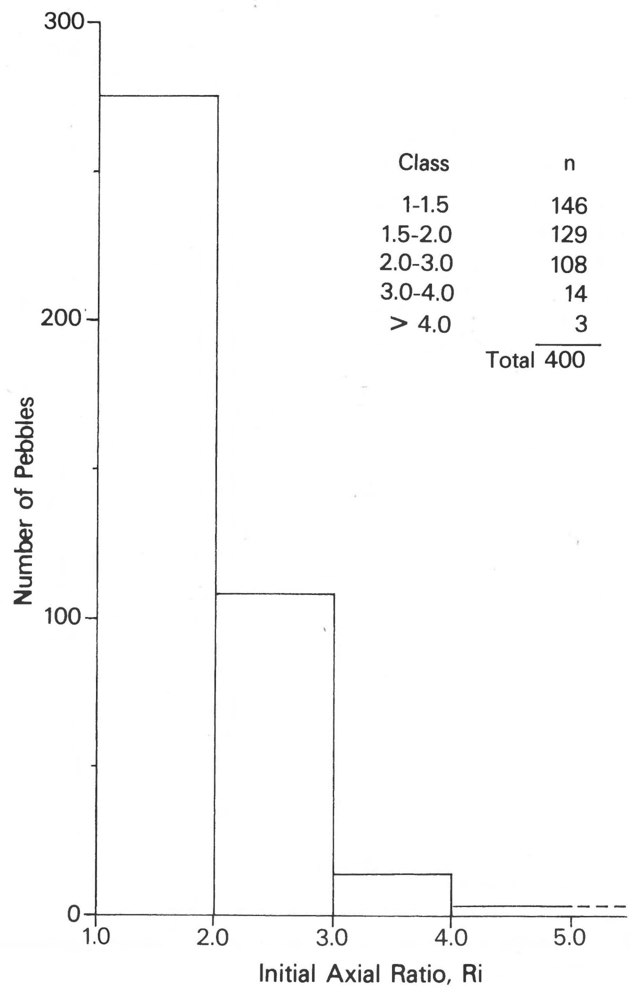



Fig. 4
The distribution of calculated initial axial ratios of 400 pebbles from a deformed conglomerate (Norra Storfjället, Sweden).

| | n | $\bar{R}$ | G | H |
|---|---|---|---|---|
| Subsample | 100 | 2.89 | 2.78 | 2.51 |
| Subsample | 100 | 2.87 | 2.57 | 2.37 |
| Subsample | 100 | 2.82 | 2.64 | 2.50 |
| Subsample | 100 | 2.74 | 2.53 | 2.43 |
| Whole sample | 400 | 2.83 | 2.63 | 2.45 |

Table 1
Calculated mean axial ratios ($\bar{R}$,G and H) of 400 deformed pebbles from a conglomerate, Norra Storfjället, Sweden.
Rf/$\phi$ method of strain analysis applied at the same outcrop gave a strain ratio of 2.40.

the initial shape of the markers (for example those of Dunnet (1969), Elliott (1970), Mathews *et al.* (1974)). Nevertheless, the harmonic mean of Rf may be a useful indication of Rs when a quickly-derivable estimate is required or when information about individual marker orientations is lacking.

The arithmetic mean of Rf is of little value for estimation of strain as it is strongly dependent on the initial shape factor.

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## REFERENCES

Barr, M.W.C. (1976) – Crustal shortening in the Zambezi Belt. Phil. Trans. R. Soc. Lond. A., 280, p. 555-567.

Cloos, E. (1947) – Oölite deformation in the South Mountain Fold, Maryland. Geol. Soc. Am. Bull., 58, p. 843-918.

Dunnet, D. (1969) – A technique of finite strain analysis using elliptical particles. Tectonophysics, 7, p. 117-136.

Dunnet, D. & A.W.B. Siddans, (1971) – Non-random sedimentary fabrics and their modification by strain. Tectonophysics, 12, p. 307-325.

Elliott, D. (1970) – Determination of finite strain and initial shape from deformed elliptical objects. Geol. Soc. Am. Bull., 81, p. 2221-2236.

Griffiths, J.C. (1967) – Scientific Method in Analysis of Sediments. McGraw-Hill, New York, 508 pp.

Helm, D.G. & A.W.B. Siddans (1971) – Deformation of a slaty, lapillar tuff in the English Lake Districts: Discussion. Geol. Soc. Am. Bull., 82, p. 523-531.

Hobson, D.M. (1971) – Deformed agglomerates near Tintagel, North Cornwall. Geol. Mag., 108, p. 383-391.

Hossack, J.R. (1968) Pebble deformation and thrusting in the Bygdin area (S. Norway). Tectonophysics, 5, p. 315-339.

Lisle, R.J. (1977) – Clastic grain shape and orientation in relation to cleavage from the Aberystwyth Grits, Wales. Tectonophysics, 39, p. 381-395.

Mathews, P.E., R.A.B. Bond & J.J. van den Berg (1974) – An algebraic method of strain analysis using elliptical markers. Tectonophysics, 24, p. 31-67.

Mukhopadhyay, D. (1973) – Strain measurements from deformed quartz grains in the slaty rocks from the Ardennes and the northern Eifel. Tectonophysics, 16, p. 279-296.

Ramsay, J.G. (1967) – Folding and Fracturing of Rocks. McGraw-Hill, New York, 568 pp.

Sanderson, D.J. (1972) – Oblique folding in South-west England. Proc. Ussher Soc., 2, p. 438-442.