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MODELS FOR THE ESTIMATION OF WORLD ORE RESERVES

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ABSTRACT

Although mineral resources are non-renewable, mineral reserves can be created by new discoveries and from already identified subeconomic resources. A probabilistic model is presented to adjust reserve estimates to changes in minimum workable grade. This model is based upon a rational frequency distribution for the dependence of tonnage on cut-off and average grade. For this purpose the straight lognormal transformation is rejected, and replaced by a logarithmic, three-parameter transformation of wider applicability, including negatively skewed distributions of iron, bauxite, and phosphate deposits.

The estimation of ore reserves occasionally serves the restricted purpose of the commercial evaluation of mining property. Yet, more often such estimates have a wider scope: up to forecasts of the adequacy to supply the future needs of a country or the world.

FORECASTING ON THE BASIS OF RESERVES

Economists tend to accept estimates of world ore reserves, as periodically presented by government agencies, as exhaustive inventories, prepared from factual data of a dominant geological nature. The effect of changing commodity prices, technological progress, and general economic factors upon the cut-off grade, the mean workable grade of ore, and hence upon the reserves are frequently overlooked. On the other hand, the present tendency is to overstress the non-renewable nature of mineral reserves. It would be more relevant to consider the flexibility with which successive reserve estimates appear to be adapted to changing economic and technological conditions.

The estimates of certain mineral reserves allow for at least changes in the metal price. This is notably the case in recent OECD reports on uranium world resources.

The common preactice of forecasting future situations of ore availability can be summarized as follows: The fine metal content (R) of a given estimate of ore reserve is divided by the current rate of production to obtain the so-called static life index ($S = R/Y_o$) in years. The next step is to introduce a factor for exponential growth ($100 \cdot r\%$). In t years the production is assumed to have increased to:

$$Y_t = Y_o(1 + r)^t$$

The cumulative production during the t years is:

$$\int_o^t Y_o\,(1 + r)^t\,dt = \frac{Y_o}{\ln(1 + r)}\,[(1 + r)^t - 1]$$

The exponential life index is defined as the number of years (T_e) in which the above cumulative production is equal to the reserve (R). This equality works out to:

$$T_e = \frac{\ln[1 + s \cdot \ln(1 + r)]}{\ln(1 + r)}$$

T_e-values for copper and bauxite, based upon recent figures, are:

	R	Y_o	S	$100 \cdot r$	T_e
Copper	393	7.5	52.4	5.2%	26.0
Bauxite	11872	73.1	162.4	8.7%	32.1

Copper: metal contents in million metric tons
Bauxite: million metric tons (about 50% Al_2O_3).

A remarkable majority of metals gives present T_e-values within the narrow range of 15 to 50 years.

Life expectancy estimates can be refined by using figures for demand, instead of for production, and by taking into account the trends in recycling.

The history of successive estimates of world reserves is revealing. The U.S. Bureau of Mines has presented the following estimates of reasonably assured world copper reserves in million metric tons of copper:
1965 ---- 192.3
1970 ---- 279.4
1973 ---- 335.7
1974 ---- 393.0

An exponential curve fit to the above estimates gives

$$\hat{R} = 1.40 \cdot e^{0.0757t}$$

in which t is the year after 1900. The coefficient of determination is 0.988. Thus, it is reassuring that copper reserves grew in the recent past at 7.86% per annum, faster than the increase in the copper production at 5.22% per annum for

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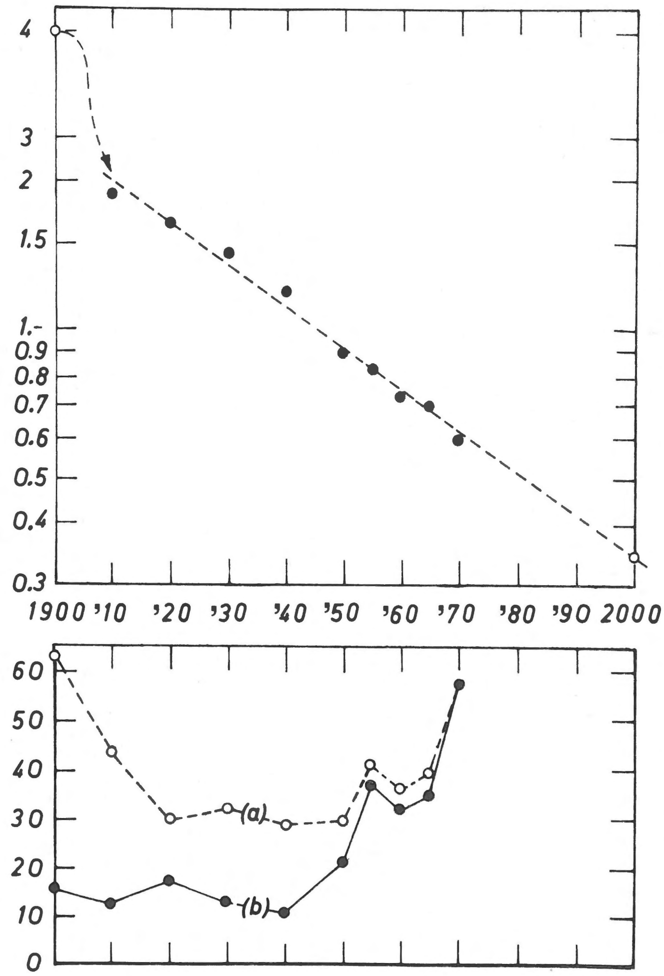

Figure 1
History of copper grade and price in the United States of America

the period 1965 – '74. Projecting these trends to the year, 2000 the exponential life index for copper would have increased in that year to $T_e = 34$ years. The growth of bauxite reserves, however, averaged during the past decade 7.5% per annum, against a corresponding growth in the production of 9.2%. In this case, exponential extrapolation shows for the year 2000 a reduction of T_e to 26 years.

Irrational as the absolute outcomes of such projections may be, the future adequacy of mineral supply depends upon the balance of the depletion record of reserves. When the growth of production exceeds the rate at which reserves are replenished, a critical situation begins to develop at the time that T_e drops below the number of years required to bring prospects into the productive stage, – a period of about 10 years. In the race against depletion of reserves, it should be kept in mind that with growing rates of production, every ton exploited will have to be replaced by more than one ton of added reserve (in terms of recoverable metal contents). The essential condition for continued future adequacy of supply is stationarity of the static life index.

Fortunately, the discovery of new mining properties is not the sole expedient to replenish the ore reserves. Most operating mines contain substantial well-known tonnages of paramarginal and submarginal grade that will eventually be assessed as ore. Fig. 1 illustrates this trend with respect to copper mining in the U.S.A. It shows the gradual decrease in the average grade of ore mined from 1910, shortly after the beginning of the "porphyry copper era", to 1970, a period over which the grade of the ore mined ell from 1.88 to 0.60 per cent copper. The exponential regression curve, fitted to this decrease in ore grade, follows an average rate of decline of 1.96% per annum. A projection to the year 2000 predicts for that year an average grade to be mined of 0.35% copper. The lower chart of Figure 1 gives average annual copper prices for the same period: (b) as direct market quotations, and (a) in constant dollars equivalent to 1970. It reveals only a minor influence of price upon the grade of the ore mined. Thus the decrease in ore grade must largely be attributed to technological progress, combined with the benefit of scale, particularly in the open-cast mining of disseminated ore deposits, as indicated for instance by a history of improving productivy in pounds of copper per man-shift.

Histories of individual mines often show a far more drastic decline in the grades of mill heads. An example is Llallagua, Bolivia's largest tin-mine. According to records published by the International Tin Council, the average grade of the ore dropped from 7.2% tin in 1929 to 0.54% tin in 1964, a rate of decline of around 7% per annum. At present the cut-off grade is about 0.15% tin and the ore as mined averages a little above 0.3% tin. During the past 50 years of continuous operations, cut-off grades and reserve estimates had to be adjusted so many times that the resident mine staff became thoroughly accustomed with life expectances for posted ore reserves that rarely exceeded a few years.

Despite the efforts of the U.S. Bureau of Mines, the U.S. Geological Survey, and other agencies to re-define and reclassify the concepts of reserves and resources, the information published on reserves remains the instantaneous picture, the snapshot, hardly adequate for long-term forecasting. It is the aim of this paper to focus more interest upon the incorporation of our growing knowledge of frequency distributions of the elements of the earth's crust in models dealing with the future sufficiency of mineral supply. Notable contributions to such an approach have come from the fields of geochemistry and geostatistics. A transformation of the variate of metal concentration is proposed for a probabilistic model of a more general application than the frequency distributions that have been favoured thusfar. The relations of tonnage and average grade of reserves at varying cut-off grades and unit-sizes of mining operations can be clarified by connecting them with a model for the frequency distribution of the grade which incorporates geochemical data, like those on the abundances of elements in the earth's crust.

The log-normal function is currently most widely used in the curve fitting to histograms of assay values of mine samples. Numerous examples can be cited of satisfactory goodness-of-fit tests to support the procedure; yet, notable exceptions have been encountered to Ahrens' "Law" of lognormal distribution of the elements in the earth's crust.

Distributions of the relatively abundant elements, including iron and aluminum, have been found to follow negatively skew distribution curves. Commercial phosphate deposits show near-symmetrical frequency distributions of BPL-values. Even some distributions of trace elements, as measured in geochemical exploration, distincly depart from a rule of log-normality.

A theoretical flaw of the log-normal model is that its range for the variate of grade extends to infinity. Perhaps more serious is the inconsistency of the model with regard to the constant sum of all constituents. When all constituents follow log-normality, they cannot add up to 100 per cent in every sampled spot. For example, a vein deposit that exclusively consists of gold and quartz, would probably show a log-normal distribution of the gold, while the complementary mineral, the quartz, will be seen to follow a different kind of distribution. The same applies to the impossibility that all constituents follow a normal distribution.

The trick has been used to reverse the log-scale for histograms with an observed negative skewness. In our example of the gold-bearing quartz vein, the complement of each SiO_2-assay would, indeed, with the transformation $z = \ln(100 - \%SiO_2)$ approximate lognormality. This practice can, however, not be generalized on account of an intermediate "no man's land". A phosphate deposit with an average grade of close to 21% P_2O_5, that is half the maximum possible grade of the pure valuable mineral (fluorapatite) is likely to produce a near-symmetrical, unimodel frequency distribution that would leave the choice to fit a positively skew log-normal curve, or a negatively skew curve for the complements of the assays. In either case a poor fit is to be expected.

The variable grade (x) of a constituant, given in per cent, parts per million, or pennyweights per ton, is invariably bound to an upper barrier (b). The upper value can be appreciably below 100 per cent. For a mineralization of chalcopyrite, the barrier for the copper grade would be that of pure $CuFeS_2$, or b = 34.5% copper.

It is proposed to substitute the log-normal fransformation z = ln x by the three-parameter transformation:

$$z = \ln \frac{b \cdot x}{b - x} \text{ where } 0 < x < b$$

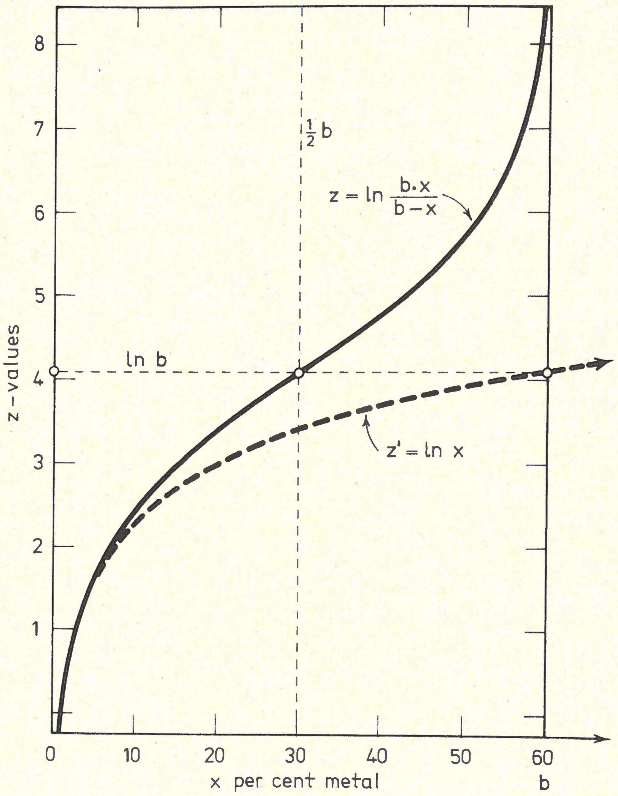

Figure 2
The universal grade transformation function

$$z = \ln \frac{b \cdot x}{b - x}$$

This function is shown in Figure 2 for b = 60%. The range of z is from $-\infty$ to $+\infty$ in accordance with that of a normal distribution. The b in the numerator of the quotient in our "universal" grade-transformation function facilitates a direct comparison with the simple z = ln x two-parameter transformation. Figure 2 shows that the universal transformation practically coincides with the log-normal model for a range of values well below $\frac{1}{2}$b. Up to $\bar{x} =! \frac{1}{2}b$ we find that mode < median < mean. For $\bar{x} = \frac{1}{2}b$ these central values coincide and the distribution becomes symmetrical. For the range $\frac{1}{2}b < \bar{x} < b$ distributions exhibit negative skewness.

Figure 3 presents examples of frequency distributions derived from the universal grade-transformation. All five curves have in common b = 60%, the standard deviation in terms of z, $\delta_z = 1.00$, and all curves enclose the same areas. THe mean values of z, postulated to be normally distributed, have been taken at 10, 20, 30, 40, and 50% for the curves A, B, C, D, and E. These values correspond to the medians of the respective curves in terms of x. Normal distributions of z give:

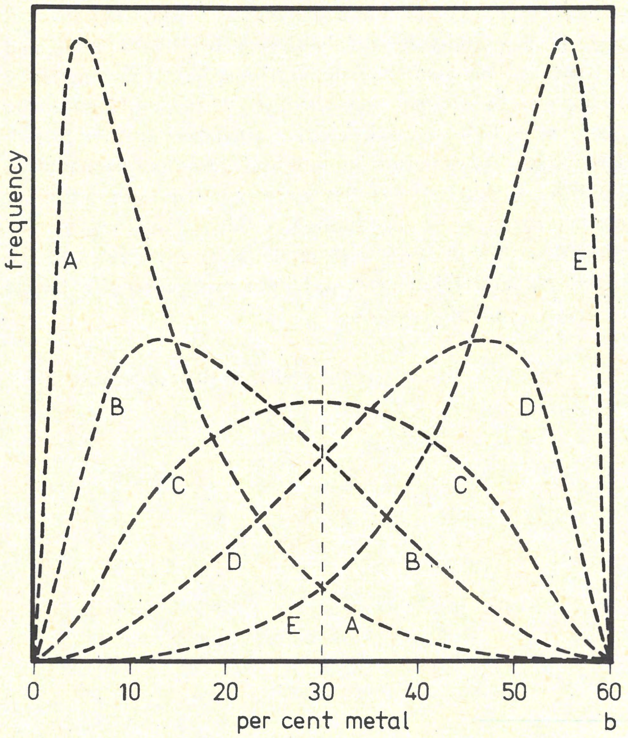

Figure 3
Examples of frequency distributions obtained with the universal grade-transformation function.

	Median	Mean	Moments	of
Curve:	grade	grade	skewness	kurtosis
A	10.–%	12.33%	+ 1.16	4.21
B	20.–	21.60	+ 0.48	2.51
C	30.–	30.00	0.00	2.16
D	40.–	38.40	– 0.48	2.51
E	50.–	47.67	– 1.16	4.21

It is worthy of notice that the symmetrical distribution C is non-normal, due to the lack of tail areas < O and < b. the low moment of kurtosis confirms the departure from normality.

As appears from the foregoing, distributions of grade depend in character of shape on the ratio of $\bar{x} : b$. This has, qualitatively, an important bearing on the economics of metal resources. In cases of prominent, positive skewness – the picture of curve A – the situation is generally one in which the cut-off grade for the ore lies well above the mode of the total mineralized environment. Hence a relatively slight decrease of the cut-off grade produces a substantial addition to the tonnage of the ore reserves. It is the situation for which Lasky in 1950 proposed his simple rule of linear relationship of average grade of reserves to the logarithm of their tonnage. This rule has fallen into disuse, but fitted fairly well to deposits of porphyry copper in the Western States of the U.S.A.

Iron mines – in particular those that exploit direct shipping ore – present the opposite case. Frequencies of iron grade display the negative skewness of our curves D or E in Figure 3. Here, mode and median being higher than the mean grade, the cut-off is likely to lie under all central values on the downslope toward zero. In that situation the lowering of the cut-off grade may result in a negligible increase of the reserve. This feature is consistent with practical experience, not only in iron mining, but also in the exploitation of bauxite. Therefore, one should question the validity of foregone conclusions drawn from computations of T_e-values, as given on page 2 of this paper for copper and bauxite. A seeming lead of bauxite over copper in the exponential life indexes will probably be more than offset by the relative scarcity of bauxite of para-marginal grade.

For the practical application of the proposed universal grade-transformation function, it may be helpful to visualize the procedure as a plot on probability graph paper of z-values against relative frequencies. Thus the curves of Figure 3 appear as a set of five paralel straight lines, intersecting the 50% frequency median at the z-values corresponding to x = 10, 20, 30, 40, and 50%. Values of z are reconverted to x with the equation:

$$\hat{x} = \frac{b \cdot e^{z}}{b + e^{z}}$$

Departure from lognormality may arise unsuspectedly when low b-values are overlooked, as could happen for beryllium in pegmatite, the BeO contents of beryl being only 14.0%. More treacherous, however, are such departures when dealing with trace elements. It has been observed that the gallium contents of bauxite follows in shape the negative skewness of the alumina distribution. The explanation is that the b-value for gallium is limited by its ionic substitution for aluminum in the host mineral, gibbsite. On the basis of Ga : Al ratios, the barrier lies here at about 140 ppm Ga.

Metal distributions in ores originating from the weathering of a proto-ore may retain some effect of the original barrier. A possible example is the distribution of nickel in the lateritic deposits of New Caledonia, in which an original limited ionic substitution of nickel in olivine has not wholly been obliterated.

Mobilized uranium, deposited in the pores of arkose sandstones on the Plateau of Colorado, reveals a superimposed barrier, caused by the available pore space, at about 4.5% U_3O_8.

Another parameter that requires attention is the variability. Coefficients of variability, determined for individual deposits, are – unfortunately – not intrinsic values, dependent as they appear to be on factors like sample spacing and regionalization.

A more promising global approach is to link cumulative frequency distributions in terms of z-values for assured ore reserves to the lower range of geochemical observation, including in particular the average grade of the element in question in the upper earth crust, its average abundance as indicated by a Clarke value, usually in ppm.

That significant correlations of metal reserves (and even of metal prices) have been established with crustal abundances supports the sense of introducing Clarke values in the field of the analysis of metal resources of the world.

Quintessentially, two points can be plotted on cumulative probability graph paper to a linear scale for transformed z-values: (1) representing known reserves above the current cut-off value, expressed as a percentage of the total mass of, say, the dry land volume to a given depth of the earth, and (2) the median grade for this 'universe' at 50% cumulative frequency. A straight line through these points could be taken as a first approximation of the desired distribution. J.W. Brinck (Formation of uranium deposits, presented to I.A.E.A., Vienna 1974) has experimented with a similar model, based on a log-binomial model, developed by the present author in 1953. This model is closely related to the lognormal model, but easier to handle over probability ranges of many standard deviations.

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