

CRITICAL PARAMETERS FOR THE PRODUCTION DEPLETION AND SUBSTITUTION OF MINERAL RESOURCES

—a horizon beyond the limits to growth —

JOHAN W. BRINCK¹⁾

SUMMARY AND CONCLUSIONS

From the mining history of different mineral commodities it is indicated that the average annual growth of production can be used as an indicator for their economic and industrial maturity.

Calculations show the growth rate to be determined by the size of the remaining reserves as function of the annual production. Its value at all times tends to remain directly proportional to the number of times that a mineral industry could double its annual production before depleting its remaining reserves.²⁾

The observations indicate logistic growth of mineral production toward a moving ceiling which is determined by both natural- (clarke and specific mineralisability) and technico-economic- (price and current state of mining technology) parameters.

If confirmed, this will enable the prediction of ultimate production levels and possible rates of substitution of scarce commodities by more abundant ones. Furthermore, it would increase the physical base for the viability of our technological society by a factor greater than 10; increasing its predictable lifetime from less than a century to well over a thousand years.

Using this relation to predict the size of the inferred reserves of a metal from its current annual production and average growth rate, close confirmation of independent estimates with the Mimic model³⁾ is obtained for the major metals iron, aluminum, copper, gold, zinc and lead, each with a value of annual production in excess of one billion US\$. Somewhat less convincing results are obtained for the almost equally well established industries of nickel, tin, mercury and antimony. They show lower growth rates than allowed by the theory. Still lower rates are found for the metals chromium, manganese and tungsten. The nonconforming metals all are commodities for which the USA territory shows marked deficiencies. Their less than normal growth seems to stress the importance of geographic and geo-political factors for the industrial development of mineral commodities, as well as their rather great substitutability.

¹⁾ P.O. Box 471. Alkmaar, The Netherlands.

²⁾ Reserves in the classical sense of estimated quantities of mineral materials in the geological environment considered economically recoverable with current technology.

³⁾ Mimic = Mining Industry Model for Inventorisation and Cost evaluation of mineral resources (Eurospectra X, 2, june 1971).

⁴⁾ Reserved and resources terminology follows the recommendations of Blondel and Lasky (1956), as modified (Commission of the European communities, 1973).

GROWTH AS AN INDICATOR OF MATURITY

From observation of the current rates of growth for different mining industries, as shown in table 1, it can be seen that the highest rates are found for elements which only recently (after \pm 1900) have become of industrial importance (Al, P, F, U, Pt).

A second group, with growth rates between 2.75% and 6.3% comprises the established mining industries whose origins often date back to antiquity. Their industrial importance generally was well assured at the beginning of this century (Fe, Cu, Zn, Pb, Ni, Mn, Mo).

In a third group, with growth rates less than 2.75% we find gold and tin, which belong to the earliest metals known and used by man. The group also contains chromium, mercury, antimony and tungsten which are metals for which the USA territory shows marked deficiencies, which may be an explanation for their lower than normal growth rates.

Apart from these exceptions, the observation seems to indicate a rather general trend of declining growth with increasing maturity of a mineral industry. This trend cannot be easily confirmed by a study of production statistics since 1945.

With few exceptions (Al, Hg, U) the often wildly fluctuating annual production figures seem to indicate more or less constant rates over extended periods. The different rates, which are typical for each industry, often seem to indicate production increases which, within a few decades, would go far beyond the possibilities offered by the demonstrated, or even the conventionally estimated inferred reserves of mineral commodities⁴⁾ (USGS. 1973).

This paradox can be explained by assuming a moving ceiling toward which this declining growth is directed at all times. Such a ceiling obviously would be determined by the natural availability of a mineral commodity on one hand, its price and the current state of mining technology on the other.

As the size of the inferred reserves of a mineral commodity, almost per definition, is determined by the same parameters, a close relation between the size of the inferred reserves and this moving ceiling can be expected.

	clarke X	spec.min. Q	price \$/kg	an.prod. P _{70/71}	growth r%	cum.prod. ΣP	dem.res. tonnes	fraction I'	inferred reserves I''	reserves I	
Al	8.1E-2	0.041	0.034	1.49E07	7.60	≈2.35E08	3.3E09	-6.3098	-6.3098	(~)	
Fe	4.8E-2	0.087	0.017	4.24E08	5.14	≈8.38E09	8.8E10	-5.2067	-5.1976	- 4.95	
P	1.0E-3	0.197	0.051	1.13E07	6.91	≈1.51E08	6.0E09	-4.6921	-4.7071	- 4.37	
Mn	1.0E-3	0.201	0.055	7.28E06	3.30	≈2.22E08	6.3E08	-5.6563	-5.8425	- 4.36	
F	6.0E-4	0.180	0.12	1.35E06	6.31	≈2.18E07	7.4E07	-5.5350	-5.5005	- 4.42	
Ni	1.0E-4	0.153	1.86	5.12E05	5.38	≈9.68E06	1.9E07	-5.1760	-5.2549	- 4.52	
Zn	8.0E-5	0.215	0.28	5.19E06	3.94	≈1.31E08	1.7E08	-4.5896	-4.4283	- 4.32	
Cr	8.0E-5	0.287	0.073	2.26E06	1.98	≈1.14E08	7.0E08	-5.2466	-5.2995	- 4.17	
Cu	7.0E-5	0.200	0.80	6.16E06	4.62	≈1.86E08	2.2E08	-4.3132	-4.4822	- 4.33	
Pb	1.6E-5	0.286	0.29	3.29E06	2.75	≈1.45E08	1.28E08	-4.2806	-4.2316	- 4.17	
U	3.0E-6	0.200	15.60	1.81E04	12.05	≈3.76E05	8.6 E05	-3.6479	-3.6480	- 4.33	prediction 1970-'85
Sn	2.2E-6	0.277	2.75	2.29E05	1.01	≈1.6 E07	4.0 E06	-4.7475	-4.7086	- 4.19	
W	1.2E-6	0.287	4.06	3.21E04	-	≈1.3 E06	1.8 E06	-5.5878	-5.5655	- 4.17	
Mo	1.1E-6	0.287	2.98	6.94E04	5.68	≈1.1 E06	4.8 E06	-4.2335	-4.0392	- 4.17	
Sb	4.5E-7	0.375	0.984	6.59E04	2.08	≈2.3 E06	3.7 E06	-4.5517	-4.4674	- 4.03	
Hg	7.0E-8	0.378	7.24	9.82E03	2.08	≈8.0 E05	2.2 E05	-4.5555	-4.4688	- 4.03	
Pt	2.8E-8	0.198	1700.00	1.18E02	9.44	≈1.4 E03	1.3 E04	-4.4887	-4.4709	- 4.37	
Au ²	3.5E-9	0.298	1126.00	1.45E03	1.49	≈7.8 E04	3.1 E04	-4.1373	-4.1535	- 4.15	revised estimate
Au ¹	1. E-9	0.354	1126.00	1.45E03	1.49	≈ (4.0 E04)		-3.5932	-3.6785	- 4.06	original Mimic estimate

≈ estimate from literature < estimate from next doubling period

I' = inferred reserves as estimated from $D \approx 100 \times r$ and expressed as the $^{10}\log$ fraction of total resources.

I'' = inferred reserves as above but corrected for difference between target price and actual long term price.

I = inferred reserves as $^{10}\log$ fraction of total resources as found with Mimic and extrapolated from data on Au¹ and U (Cu).

Source: US Bureau of Mines (14), Gocht, W (15) e.o.

table 1 a horizon beyond the limits to growth

ESTIMATING RESOURCES FROM GROWTH

In order to estimate the size of the resources which at any time determine the production level of a mineral commodity, it is assumed that the growth rate (r) is directly proportional to the number of times (D) that the mining industry could double its current production capacity before depleting the thus indicated resources, as follows:

$$r' = D = \frac{\ln(1 + (I - \Sigma P) \times \ln(1+r)/P_y)}{\ln(2)} \quad [1]$$

r' = new growth rate in percent

I = inferred resources

ΣP = former production

P_y = annual production for the reference year y

From [1] the size of the required resources for the indicated growth rate can be estimated from current production.

For comparison with estimates made at different times, the former production should be added. Thus:

$$I = \frac{P_y(2^{100r} - 1)}{\ln(1+r)} + \Sigma P \quad [2]$$

In order to make the resource estimate comparable with estimates for other mineral commodities, the size of the resource will be expressed as a fraction of the total resources (M) in the accessible part of the geological environment (R = resource base). Total resources are:

$$M = R \times X \quad [3]$$

X = the clarke value⁵⁾ (or the average concentration of the commodity in the environment)

Thus, the inferred resources, expressed as a logarithmic fraction of the total resources are:

$$I' = ^{10}\log(I/(R \times X)) \quad [4]$$

5) The clarke is the average concentration of an element in the upper part of the continental earth crust.

For the majority of mineral resources this geological environment is the upper part of the continental earth crust to a depth of some 2,5 km, containing roughly 1.0E18 tonnes of rock. Thus:

$$R = 1.0 \times 10^{18} \text{ tonnes of rock}$$

which value will be used for all subsequent calculations.

ESTIMATING RESOURCES FROM THEIR GEOCHEMICAL DISTRIBUTION

Assuming a log-binomial distribution of mineral concentrations in the geological environment, resources of different size-grade specifications can be estimated from the following general expression (B r i n c k, 1974):

$$M = R \times X = \sum_{k=0}^{\alpha} \left(\frac{\binom{\alpha}{k}}{2^{\alpha}} \times R \times X \times (1+Q)^{\alpha-k} \times (1-Q)^k \right)$$

In which [5]

R, X and M as before

Q = specific mineralisability ($0 < Q < 1$)

α = order of subdivision of the environment when the number of deposits of size s in the environment of size R is 2^{α} .

k = an integer 1, 2, 3...N ($\alpha - 1 < N \leq \alpha$)

The average concentration (X) measures the absolute abundance of a mineral commodity; the specific mineralisability (Q) its dispersion in the environment.

These functions can be easily grasped from the expressions:

$$\gamma = X \times (1 - Q^2)^{0.5\alpha} \quad [6]$$

γ = median concentration for a subdivision of the order α

$$\sigma = 0.5\sqrt{\alpha} \times \ln\left(\frac{1+Q}{1-Q}\right) \quad [7]$$

σ = standard deviation

The standardised variable z, which measures the deviation from the median in units of the standard deviation, in the log-binomial distribution has a maximum value which is determined by the order α , as follows:

$$z_{\max} = \sqrt{\alpha} \quad [8]$$

The corresponding probability of occurrence of a deviation equal or greater than z_{\max} :

$$P_z = \frac{1}{2^{\alpha}} \quad [8a]$$

The maximum concentration (x_{\max}) corresponding to z_{\max} :

$$x_{\max} = X \times (1+Q)^{\alpha} = \gamma \times \left(\frac{1+Q}{1-Q}\right)^{\frac{1}{2}\alpha} = e^{\ln \gamma + z_{\max} \times \sigma} \quad [9]$$

Economic mineral deposits, whether metalliferous or non-metallic, are extreme accumulations or concentrations of one or more useful substances that are for the most part sparsely distributed in the earth's outer crust (5). Such deposits can be divided into:

giant	deposits	> 1×10^9	tonnes ore	($\alpha < 30$)
large		$3 \times 10^7 - 1 \times 10^9$		($30 < \alpha < 35$)
medium		$1 \times 10^6 - 3 \times 10^7$		($35 < \alpha < 40$)
small		< 1×10^6		($\alpha > 40$)

For high Q-values (> 0.25) the ore deposits of an element are very unevenly distributed over the world and ore reserves are concentrated in few mineral districts or provinces. The contrast between ore grade mineralisation and common rock is pronounced and concentration factors ≥ 1000 are the rule (Cr, Pb, Sn, W, Mo, Sb, Hg, Au).

For intermediate values of Q (0.15 – 0.25) such concentration factors in ore are between 100 – 1000 (P, Mn, F, Ni, Zn, Cu, U, Pt).

For lower Q-values the concentration factors are much lower (Al, Fe).

If we consider the accessible part of the earth's crust as one large mineral deposit there appears to be a clear advantage in selective mining of commodities with high Q-values.

Disregarding for one moment the environmental aspects or the very important cost savings which can be obtained from scale economies, the major advantage of mining rock with a 1000-fold concentration instead of ordinary country rock, is the lower cost of extraction per unit of contained metal, by a more or less equal factor.

The marketable mining products for commodities with a high Q-value can be produced at lower cost than those of equally abundant commodities with lower Q-value. This relation is expressed in their average long term price as follows:

$$P_t = \frac{e^{aQ+b}}{X} \quad [10]$$

P_t = target price for a mineral commodity as contained in its marketable mining product (ore, concentrate, metal)
a, b and e are constants ($e = 2.71828$)

Expressed in 1970 US\$ per kg element contained, the following values for a and b were determined from X and Q of 15 different mineral commodities and their average long-term price (P_1) over the period 1956 – 1970.

6) This indicated probability differs significantly from the probability as found for equal z-value from the error-function (normal- or Gaussian distribution)

$$a = -25.936498$$

$$b = -4.749741$$

and

$$P_t = 1.028 \times P_1 \pm 13.71\% \quad [11]$$

for 99% confidence limits

$$0.68P_1 < P_t < 1.55P_1 \quad [12]$$

The expression [10] indicates an average ore value ($Q = 0$; $X = 1$) of US\$ 8.65 per tonne, which seems to be the right order of magnitude for industry-wide exploitation cost of mineral commodities during the reference period⁷⁾.

Thus, for elements with equal Q -values, the long range price of their mining product is inversely proportional to their average abundance in the earth's crust.

On the other hand, for equally abundant elements with different Q -values, the one with the higher value will be cheaper than the one with the lower value for the specific mineralisability.

For equal size (α) and rarity (z) of their ore deposits, the amount of element concentrated in such ore deposits, expressed as a fraction of its total resources will be greatest for the element with the highest Q -value, as can be easily checked from [6] and [7].

Defining "inferred" reserves of an element as the fraction of its resources that with current technology could be extracted from the environment at costs equal or less than the thus indicated target price, the following relation is indicated:

$$I = {}^{10}\log(cQ + d) \quad [13]$$

With the help of the Mimic model (B r i n c k, 1974) the following values for c and d were determined from the size of the inferred reserves for gold ($Au' - Q = 0.354729$; $I = -4.059982$) and uranium ($U - Q = 0.200268$; $I = -4.363178$).

$$c = 2.83 \times 10^{-4} \quad d = -1.34 \times 10^{-5}$$

For very low Q -values the size of the inferred reserves (I''') may be zero or negative as can be seen from:

$$I''' = (2.83 \times 10^{-4}Q - 1.34 \times 10^{-5}) \times R \times X \quad [14]$$

for $Q \leq 0.047$

Here, the rare "ore" grade mineralisation would be only slightly better than the average grade of the environment. For such low enrichments the exploration costs plus royalties easily could equal or surpass the economic advantages for mining such higher grade material. The notion of reserves in the economic sense is bound to disappear even if a rest value for such higher grade concentrations could remain. The

production costs for the marketable product, however, would be determined exclusively by the marginal cost of its extraction from average rock and the viability of its mining industry by its competitiveness with other mineral commodities (substitution) or other sources of supply such as the hydrosphere (Mg), atmosphere (O, N), the ocean bottom (Mn-nodules) etc.

At their average grade, the resources for most commodities are almost infinitely large ($X \times R$). Obviously, their extraction would be uneconomic in view of their marginal utility at the marginal cost of production from such average grade materials.

Some complications arise for elements with high clark values (O, Si, Al, Fe, Mg, Mn, Ca, K, Na, Ti, P etc) or, more in general, for which the combined specific mineralisability and clark indicate "ore" grades approaching, equalling or exceeding unity or the concentration of the element in its common ore minerals.

The physical impossibility of enrichment beyond such barriers leads to higher frequencies of occurrence of concentrations near this barrier grade than predicted from the log-binomial distribution of the grade variable alone. This can be easily verified from the common occurrence of major elements in concentrations approaching this barrier grade (Ca in limestone, Si in quartz sands, Al in bauxites and some clays, Na in rock salt etc.). A mathematical expression for this behaviour has been worked out in an advanced version of the Mimic model (De W i j s, 1975) and can be found from the transformation of the grade variable (X_i) into the variable Z , as proposed by De Wijs (1975):

$$Z = \ln \frac{B \times X_i}{B - X_i} \quad [15]$$

in which

B = barrier grade.

With this transformation the boundaries for the validity of the log-binomial distribution of element concentrations, as determined by grade and size, are fixed by the average grain size of the minerals making up the environment on one side, the whole-rock grade of the concentrations on the other.

The grades of the concentrations then logistically approach this barrier grade, the impossibility of higher enrichment being compensated by the larger size (higher frequency of occurrence) of such concentrations.

For ore grades up to 1% of this barrier grade the frequency distributions of the grade variable are almost equal and the size of the inferred reserves from expression [13] for the elements gold and uranium would be valid for both interpretations. The estimated Q -values however, may be subject to a slight, but systematic underestimation which generally would fall well within the range of the probable error of such estimates.

7) according to Goeller and Weinberg the average value for total mineral production in 1968, inclusive fossil fuels and building materials can be estimated at \$ 9.16 per ton (6).

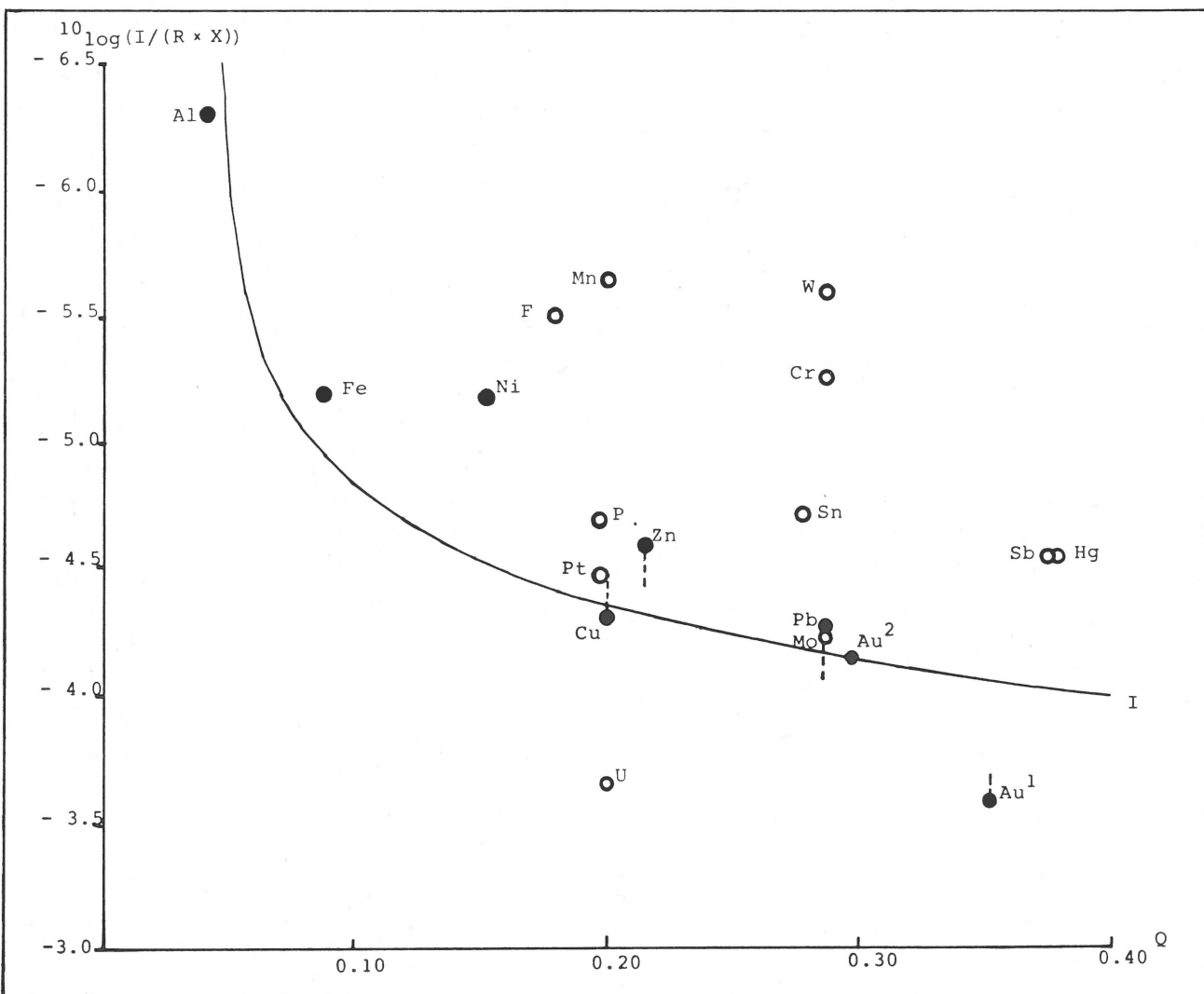


Figure 1 Inferred reserves from Q and from $r \% = D$

COMPARING THE RESOURCES ESTIMATES

metal

estimated value
of production
(billion US\$)

In figure 1, the inferred resources (I') from growth are plotted as a function of Q against the inferred reserves (I).

The figure shows a remarkable correlation between the size of the inferred resources from growth and the inferred reserves for the major metals. Major metals here are defined as metals for which the value of their mining product exceeded 1 billion US\$ in 1969/70.

iron	28
copper	6
aluminum	6
gold	2
zinc	2
nickel	1
lead	1

Metals with annual value of mine production between 0.1 – 1 billion US\$, in decreasing order of importance were tin, silver, manganese, uranium, molybdenum, the metals of the platinum group, tungsten, magnesium, titanium, chromium, mercury and antimony. Valuedwise, the non-metallic commodities phosphor and fluorine also belong to this group.

Without going into a detailed analysis of the meaning of this correlation for the individual commodities, some general observations, indicative for possible systematic estimating errors can be made.

Inferred resources from growth are equal or greater than the inferred reserves for three of the seven major metals. Assuming average economic parameters for the estimate of the inferred resources by the Mimic model, this result would have been statistically predictable as highly probable if the assumed relation between annual growth rate and the size of the inferred reserves should be valid.

However, some contributing factors may further explain why the major metals show the observed behaviour.

For aluminum with its high clarke value, a systematic underestimation of the Q-value would have its largest effect. Its correction would tend to shift the inferred resources further toward the inferred reserves.

For iron, the estimate of its former production has been made from the size of production during the following doubling period, assuming exponential growth of production at the current growth rate:

$$\Sigma P \leq P_0(1 + \ln(1 + r))/\ln(1 + r) \quad [16]$$

It is indicated from other metals with a long mining history that the production statistics sometimes show larger size of former production than found from this expression. Previous production of iron therefore could be underestimated. Its correction again would tend to shift the inferred resources toward the inferred reserves.

For nickel a rather large deviation is found for the size of the inferred resources from growth. There appear to be no obvious estimating errors to explain away this deviation, which is of the same order as for tin, antimony and mercury. Together with manganese, tungsten and chromium these metals apparently have significantly lower growth rates than would be possible from the size of their inferred reserves. Apart from the particular, almost monopolistic industrial structure for some of these metals and the international price agreements for tin, these commodities seem to have in common that their important reserves are found outside the USA territory, which itself shows marked deficiencies for these metals.

The comparatively too large inferred resources for uranium are based on a predicted growth rate of 12.05% for the period 1970 – 1985 (9). The military demand until 1959 and the much smaller civilian demand in 1970 would have indicated a negative growth for the reference period. Therefore the estimate is not really comparable with the other estimates. The Au¹ estimate was based on a clarke value of

1.0E-9 (1ppb). The revised estimate Au² is based on a more generally accepted clarke value of 3.5E-9 (3.5ppb) and the long term average grade and size of its ore deposits.

THE CORRECTING INFLUENCE OF PRICE

For a quantification of price as an indicator and correcting factor for anomalous behaviour of a mineral industry we should return to the interesting expression [10].

The expression relates the average long term price of a mineral commodity to its natural parameters clarke and specific mineralisability, apparently independent from the size and degree of depletion of its reserves.

This implies a high degree of substitutability for mineral resources. The substitution of relatively scarce (expensive) commodities by more abundant ones until 1970 has prevented extreme long term price increases over the target price for any particular commodity.

It does not explain the frequent and often brutal short term price fluctuations in the mineral industry. The latter can be attributed to disturbances of the balance between supply and demand. The causes for such disturbances could be either internal, such as an unbalanced development of demonstrated reserves and production capacities for the actual or predicted demand, or, they could be external, being determined by politico-economic actions. They all will tend to change one or more of the parameters that determine the normal balanced growth of a mineral industry.

By defining balanced development of a mineral industry as logistic growth of its production toward a ceiling which, for current technico-economic development always is determined by the clarke and specific mineralisability, the correcting influence of price could be estimated from expression [10].

For a fixed value of Q, as in figure 1, the equilibrium price would be inversely proportional to the clarke value. A price increase over the target price therefore could result in an annual growth rate as if the inferred resources (as function of an apparent clarke value) would be greater by a proportional amount and vice versa.

In order to evaluate its importance, this correction should be applied to the inferred resources from growth (I'), as follows:

$$I'' = I' + {}^{10}\log \frac{P_t}{1.0281 P_1} \quad [17]$$

The correction would shift the inferred resources for zinc and gold toward the inferred reserves, but would over-compensate for copper and molybdenum as indicated from fig. 1 (dashed lines).

Thus, either with or without this price correction which seems to fall well within the range of the probable error of the estimates, there appears to be a very strong case for accepting this theory of logistic growth of mineral produc

tion toward a moving ceiling which is determined by the clark and specific mineralisability of a mineral commodity on one hand, its price and the state of technology on the other. The size of the resources determining this production ceiling is determined by the size of the inferred reserves and can be found from [14].

A TIME FRAME FOR THE DEPLETION AND SUBSTITUTION OF MINERAL RESOURCES

Accepting logistic growth of mineral production we will assume that annual production of less than 1/10 000 part of the inferred reserves would indicate a pre-industrial stage for a mineral commodity. Thus, a unit of annual production (U_p) could be defined as:

$$U_p = 1.0 \times 10^{-4} I''' \quad [18]$$

For $P_0 = \Sigma P = U_p$; $I = I'''$ and $r = 1.00$ in expression [1] a growth rate, annual production and cumulative production can be calculated for the year 0 + 1 and so on (see table 2).

From an original growth rate of 12.76% and annual production of 1.13 U_p in the first year the growth rate after 30 years will have diminished to 6% for an annual production of 9.8 U_p .

After another 30 years the growth rate will have decreased to slightly over 3% and annual production will have increased to 36.7 U_p . A third period of 30 years ends up with growth of 1.3% and annual production of 70 U_p . After some 130 years, when zero growth is reached, the depletion of the inferred reserves is still 50 years away for an annual production rate of 85 U_p , or 0.85% of the size of the originally existing inferred reserves, indicating a predictable lifetime for inferred reserves in the order of 180 years. This would be the prediction for the possible development of a new metal, provided that its marginal utility per dollar would match its target price as determined from X and Q for optimum development under current technology. Beyond the inferred reserves there would be potential reserves which could be exploited at costs less than 2 – 3 times the current prices and which at the indicated zero growth would last for at least another 1000 years. For current technology this would indicate an average annual cost increase of less than 0.1%. This is far below the current average cost savings from technological improvements which have been estimated between 0.5 and 1 percent annually (M a b i l e, 1968)

The difference between cost savings through technological improvement and cost increases as a result of the progressive depletion of mineral resources is behind the explanation for the different, but almost constant growth rates of mineral commodities over extended periods of time.

It allows us to bring the actual historical development of a commodity, which may span thousands of years, inside the rather limiting time frame of logistic growth. The unit of production has been much smaller in the past and, judging from all available evidence until 1970, could be larger in the future.

The fact that such old mineral industries do profit from current technology, however, enables us to estimate their apparent age, expressed in years for current technology, as function of their growth rate, the size of their annual production or their depletion as indicated by cumulative production (see table 2).

From the different possible combinations to estimate the apparent maturity of a mineral industry, the estimates from current annual production and cumulative production appear to be the most interesting as they show a systematic deviation from the predicted behaviour for the major metals (fig. 2). Observed production is 0.68 ± 0.04 times the theoretical capacity and indicates depletion of the reserves after 116 – 130 years or, at the onset of zero-growth. For growing total reserves of the mineral industry, the growth of the individual commodities appears to become stabilised at different rates. As scale economies obviously are the major contributor to overall reserve increases, this can be explained from the different sensitivity of individual industries to scale economies. It indicates, through differences in growth rate, a gradual shift from and substitution of the relatively rare high-Q, by the generally more abundant low-Q commodities.

The size of the inferred reserves of the mineral industry as a whole grows at a lower average rate than the depletion of

YEAR	100K	P UNITS	S UNITS	LOG S
0	100.00	1.00E+00	1.00E+00	.000
1	12.76	1.13E+00	2.13E+00	.328
5	9.24	1.63E+00	7.84E+00	.894
10	8.51	2.48E+00	1.84E+01	1.265
15	7.82	3.66E+00	3.42E+01	1.534
20	7.18	5.23E+00	5.70E+01	1.756
25	6.57	7.27E+00	8.91E+01	1.950
30	6.00	9.84E+00	1.33E+02	2.124
35	5.47	1.30E+01	1.91E+02	2.282
40	4.97	1.67E+01	2.67E+02	2.427
45	4.50	2.10E+01	3.63E+02	2.560
50	4.05	2.58E+01	4.82E+02	2.683
55	3.64	3.11E+01	6.27E+02	2.797
60	3.24	3.67E+01	7.99E+02	2.903
65	2.87	4.26E+01	1.00E+03	3.000
70	2.53	4.86E+01	1.23E+03	3.091
75	2.20	5.46E+01	1.49E+03	3.174
80	1.89	6.03E+01	1.78E+03	3.251
85	1.60	6.56E+01	2.10E+03	3.322
90	1.32	7.04E+01	2.44E+03	3.388
95	1.06	7.46E+01	2.81E+03	3.448
100	.82	7.81E+01	3.19E+03	3.504
105	.60	8.09E+01	3.59E+03	3.555
110	.41	8.28E+01	4.00E+03	3.602
115	.25	8.41E+01	4.42E+03	3.645
120	.12	8.48E+01	4.84E+03	3.685
125	.05	8.51E+01	5.27E+03	3.722
130	.01	8.52E+01	5.69E+03	3.755
135	.00	8.53E+01	6.12E+03	3.787
140	.00	8.53E+01	6.55E+03	3.816
145	.00	8.53E+01	6.97E+03	3.843
150	.00	8.53E+01	7.40E+03	3.869
155	.00	8.53E+01	7.83E+03	3.894
160	.00	8.53E+01	8.25E+03	3.917
165	.00	8.53E+01	8.68E+03	3.938
170	.00	8.53E+01	9.10E+03	3.959
175	.00	8.53E+01	9.53E+03	3.979
180	.00	8.53E+01	9.96E+03	3.998
181	.00	8.53E+01	1.00E+04	4.002

$$D = \text{LOG}(1.0 + (1-S) * \text{LOG}(1+K)/P) / \text{LOG}(2)$$

table 2

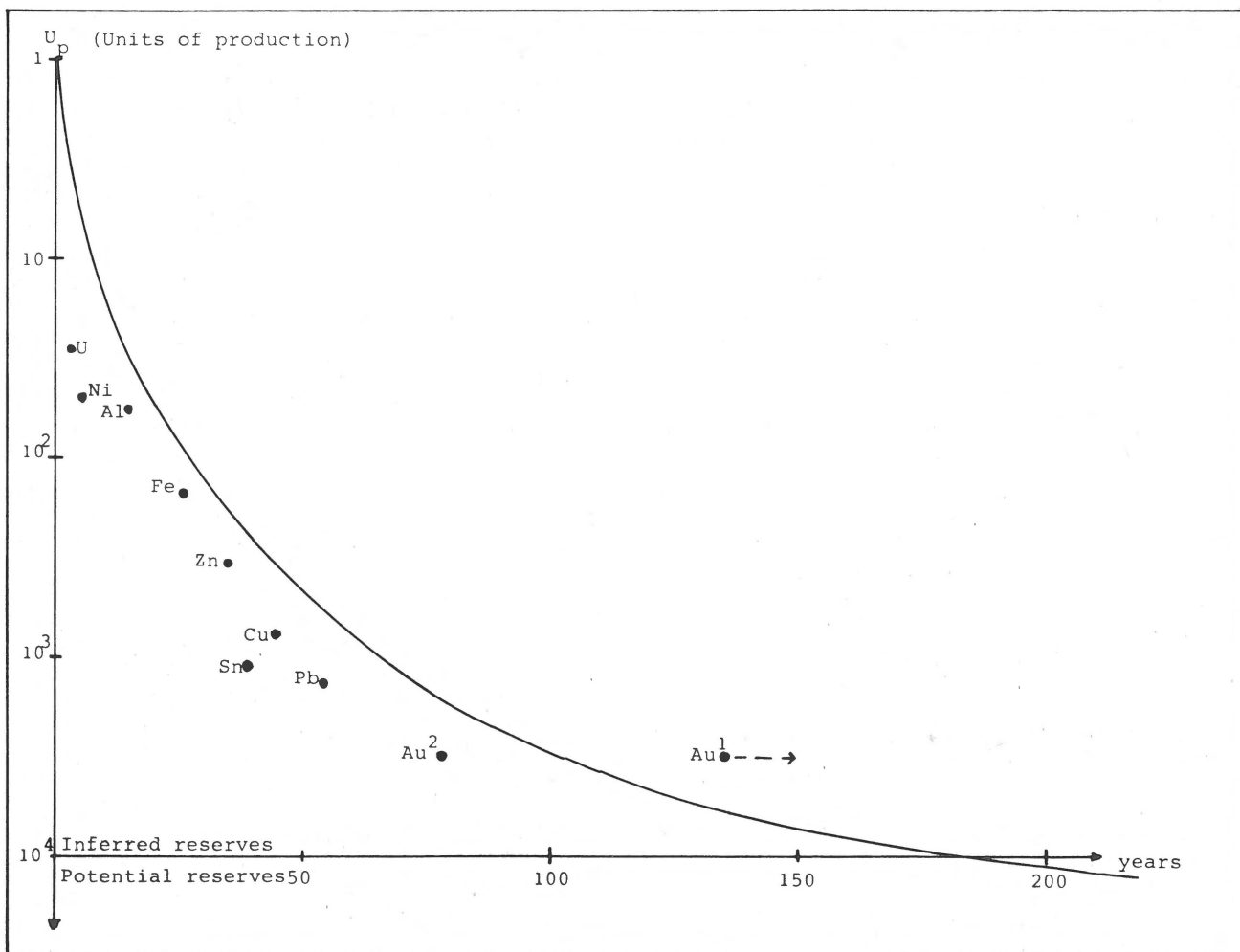


figure 2 Maturity of mineral industries from depletion and annual production

the individual commodities. Therefore, this depletion will have an effect on the dynamics of the world system, which probably can be best described by "aging".

For positive growth of the inferred reserves, for which the rate is determined as a function of technology, availability of energy, environmental-, political- and social-economic restrictions, this aging will be slower than indicated by the time scale of the logistic model.

For zero-growth of the inferred reserves, the individual commodities would reach this stage according to their apparent age. The system as a whole then still could survive at slightly increasing cost of mineral supply for at least another 1000 years.

An actual decrease of the size of the inferred reserves from such external factors probably would result in violent price increases and economic stagnation until a new equilibrium within this timeframe of logistic growth will be reached, obviously at a lower level of material prosperity.

THE INTERNAL PARAMETERS FOR THE PRODUCTION, DEPLETION AND SUBSTITUTION OF MINERAL COMMODITIES

Apart from the external parameters that regulate the size of the inferred reserves of mineral commodities as a function of marginal utility per dollar versus marginal costs, there appear to be only three internal parameters that determine the production potential of the individual commodities within the time frame of logistic growth.

The Clarke and specific mineralisability determine absolute abundance and dispersion in the geological environment. These in turn determine the cost differences between the mining products of different mineral commodities.

The cumulative production (ΣP) determines the growth rate of annual production as a function of the size of the inferred reserves of a mineral commodity.

This observation is valid for the major metals for which the size of the inferred reserves appears to be determined almost exclusively from the average cost of exploration and exploitation as well as for the commodities which show lower growth rates and production levels than indicated from these parameters. Within the economic context of these industries the external parameters apparently impose more stringent restrictions on their growth than for the major metals. Such restrictions can be of technical nature such as a lack of applications or insufficiency of metallurgical knowledge to make the commodity competitive with others. They also may be economic, geographic or geo-political. For the long range, however, they all are within human control.

The best long term indicator for the possible growth and production potential of a mineral commodity therefore appears to be its apparent maturity, as indicated from its current annual production (P_y), expressed in units of production (U_p).

For the short term, the exponential life index (de Wijs, 1975) which expresses the size of the demonstrated reserves as a number of years production at the current growth rate, still gives the best indication to regulate exploration activities.

In fig. 3 an attempt is made to visualise these rather complicated relations in a comprehensible form.

Total resources, expressed in $10 \log$ tonnes metal are plotted against their clarke values.

For a number of metals the remaining reserves are plotted on this diagram. They are divided into demonstrated reserves (full lines) and inferred reserves (dashed). The size of the inferred reserves as indicated from their clarke and specific mineralisability is indicated and can be compared with the average size of inferred reserves as found by Erickson (1973) by extrapolating from USA lead reserves (0.004% of total resources).

Separately the size of the demonstrated reserves of these metals is superimposed on their remaining reserves (dots).

For mature industries such as gold, mercury, silver, tin and lead it can be seen that the demonstrated reserves are smaller than the cumulative production until 1970. For such industries the growth potential appears to be relatively low.

For the younger industries the dots fall within the range of the demonstrated reserves, indicating a relatively important growth potential. For some very young industries, such as platinum, thorium, titanium, iron and aluminium, cumulative production is practically negligible in view of production- and growth potential, even at zero growth for the size of the inferred reserves.

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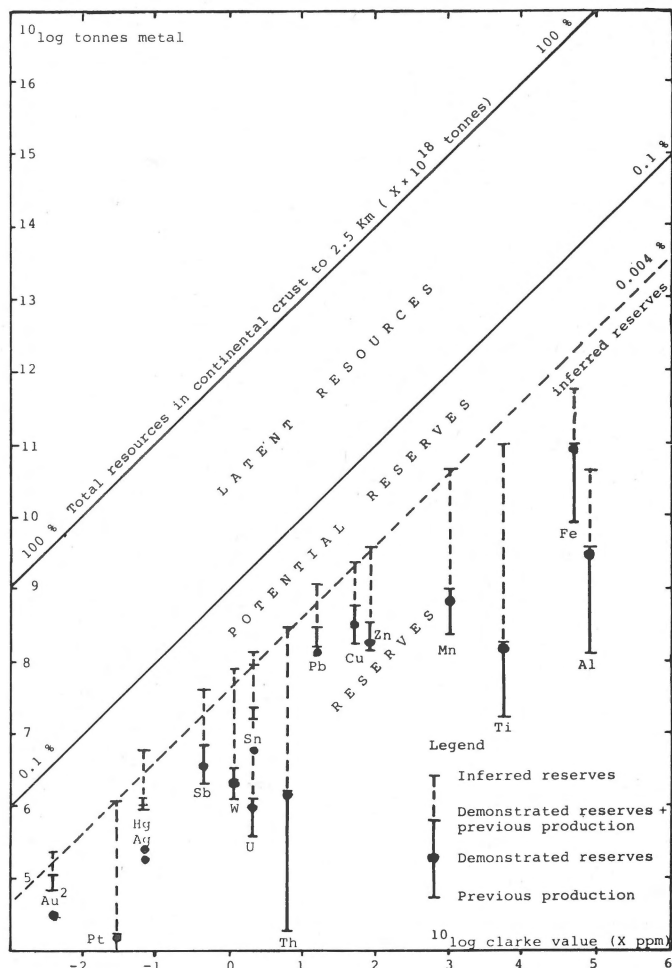


figure 3

Directions for Science and Technology" under the chairmanship of Dennis Gabor and Umberto Colombo (in press). Of particular interest were many conversations with Cesare Marchetti of the International Institute for Applied Systems Analysis (IIASA). The described relations appear to be at the base for an explanation of the very stable rates of market penetration and substitution of the energy resources coal, petroleum and natural gas as observed and described by him (1975). The direct inducement has been a joint study for the "Stichting Toekomstbeeld der Techniek" with Henri J. De Wijs, who has been so kind to critically read this paper. His criticism and valuable suggestions have contributed much to its presentation.

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