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A THREE DIMENSIONAL VECTOR METHOD AS AN AID TO CONTINUOUS-DIPMETER INTERPRETATION

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ABSTRACT

Means of dipmeter readings derived by a three dimensional vector method have proven to be useful in deriving structural dip and azimuth from a collection of widely scattered dips. A computer program to calculate 3-D vector means and related statistics has been developed.

Dip distributions are comparable to the spherical normal distribution so that for practical purposes a measure of reliability of the vector means can be calculated. The 95% confidence cone around the vector mean is chosen as the most convenient yard-stick. Confidence angles for the mean dip angle and mean azimuth are derived separately from this confidence cone.

Preferably, study and correlation of well logs should precede the averaging procedure. This enables the selection of intervals, which contain readings more-or-less conforming to the structural dip and azimuth.

The calculated average dips can be displayed on a tadpole plot, or other plots against depth. The average dips are much easier to interpret than the original data, especially when using a condensed depth scale.

The difference between successive vector means is a measure of the rate of change of dip with depth and can reveal the presence of discontinuities, such as faults and unconformities which might otherwise remain undetected in the "raw" data.

The approach is applicable to both 3-arm dipmeter logs and the more modern 4-arm dipmeter logs for which the digitized curves are correlated by the computer.

INTRODUCTION

Structural information on the attitude of subsurface strata can usually be obtained by running the Continuous Dipmeter tool or a similar device in uncased boreholes. The downhole instrument has three (or four) pads containing a measuring electrode. The pads ride along the borehole wall and are spaced 120° apart (or 90° in the case of the four-arm dipmeter) in a plane approximately perpendicular to the axis of the borehole.

One very detailed resistivity curve reflecting lithological change with depth is recorded through each of the electrodes. Correlatable events on the resulting set of three (or four) curves can be interpreted as planes intersecting the borehole (fig. 1).

From the relative depth displacements of the events on the curves and knowledge of the attitude of the measuring sonde and that of the borehole, the dip-direction and dip angle ("dip") of such planes can be calculated.

This results, with modern instrumentation, in many hundreds, sometimes thousands of dips recorded in one borehole. Usually the results are presented graphically in a "tadpole" plot showing amount of dip versus depth, the dip direction being indicated by small arrow-like signs (fig. 2).

Dipmeter interpretation using a tadpole plot has become increasingly difficult with successive improvements in the dipmeter tool or calculation procedure. The wealth of data makes it hard to estimate average structural dips, especially where the average dip angle is low.

Clearly, the profusion of dips calls for a statistical treatment of the orientation data to obtain a mean azimuth and dip angle and a measure of the uncertainty of these statistics.

Several graphical, basically two-dimensional methods are being widely employed, such as the use of Schlumberger's Schmidt and Polar diagrams or the graphs plotting azimuth and dip angle separately

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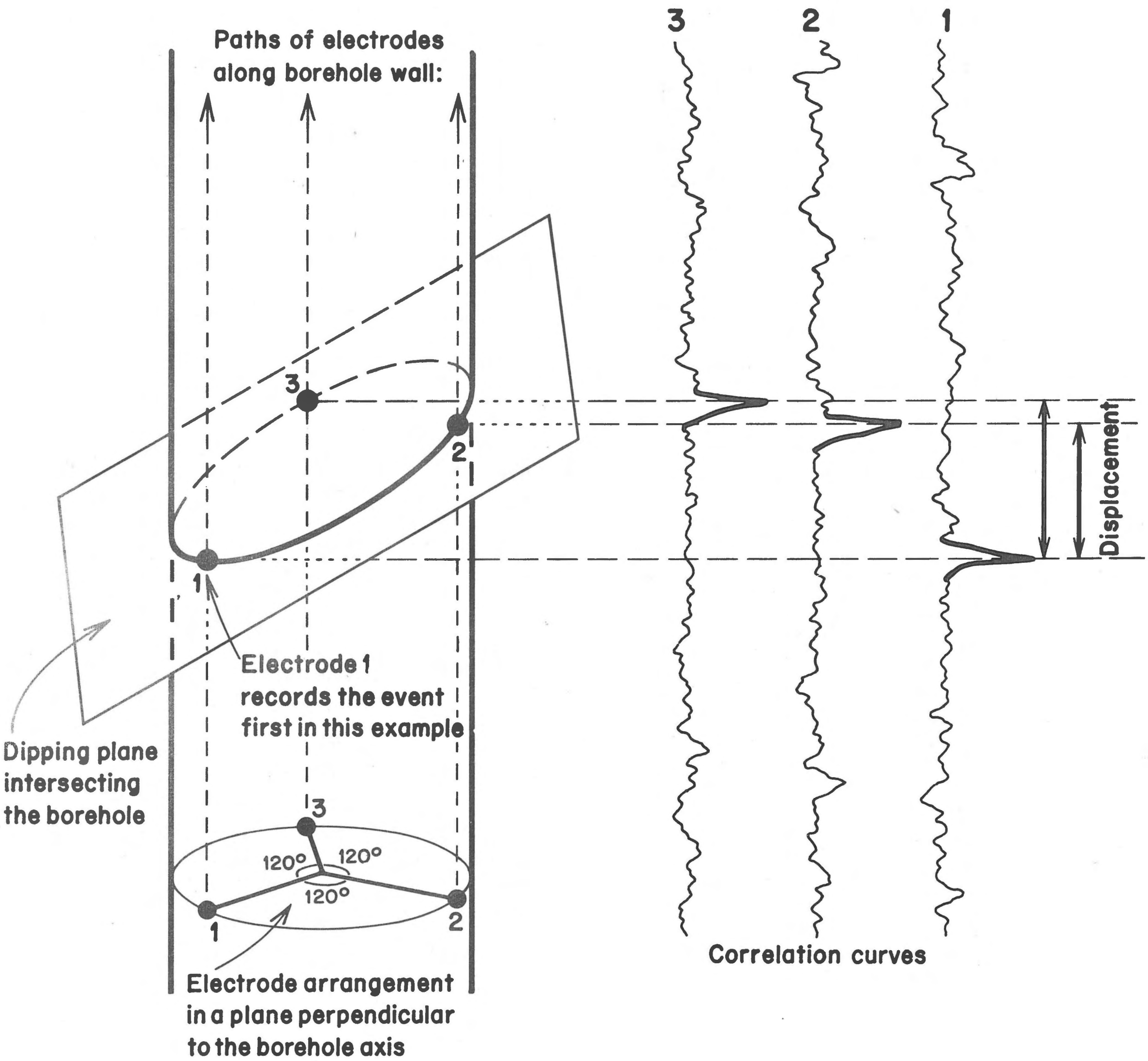


Fig. 1
Principle of the continuous dipmeter.

against depth. Under favourable circumstances the mean dip angle and azimuth can be estimated from such plots.

The disadvantage of these methods is that they are only reasonably efficient when least needed, i.e. it is easy to assess the mean azimuth and dip angle only if the mean dip angle is large and the amount of scatter small. In such cases the usual tadpole plot as supplied by Schlumberger is generally good enough for this purpose.

The inadequacies of a two dimensional representation can be demonstrated by the following example of dips from a thinly bedded limestone interval shown on a tadpole plot (fig. 2).

On the left, the figure shows the data as observed. The data shown on right have been obtained by rotating the set of orginal data in space in such a way that the mean dip angle was reduced from 21° to 1°, but the mean azimuth and three-dimensional measure of scatter has been maintained.

The resulting apparent increase in scatter on the tadpole plot is due entirely to the method of presentation.

A sound statistical treatment can only be obtained

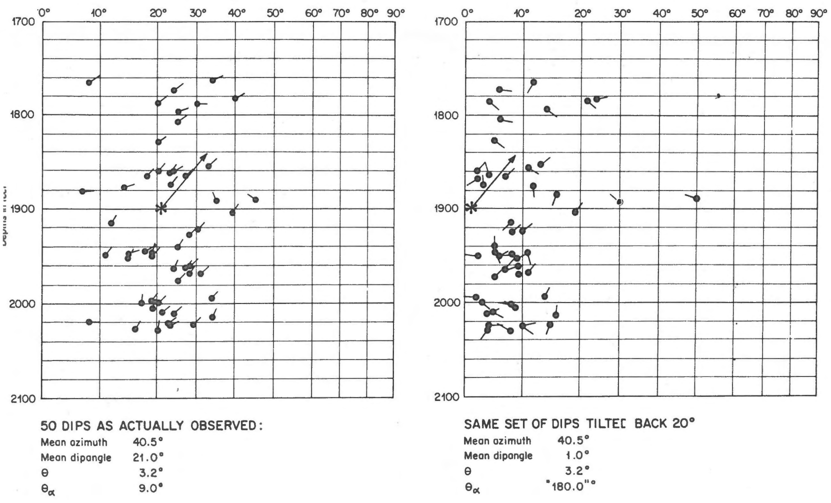


Fig. 2
Limitation of 2-D representation.
Low dip angle.

by recognizing that the dipmeter supplies three dimensional orientation data. A two-dimensional approach to analyse such data leads to difficulties in the majority of cases.

Therefore a three dimensional vector method, as used normally for palaeomagnetic data, was chosen and programmed in FORTRAN to treat the numerical dipmeter data which accompany the tadpole plots and can be obtained in punched card form. The aim of this paper is to discuss the principles of the method rather than the computer program.

The practical application of this statistical method poses some problems such as selection of data, depth intervals for which dips are to be averaged, exclusion of fault drag zones and presentation of results. These aspects are discussed in the light of the experience gained during the last year using this method over a wide variety of geological conditions.

AVERAGING PROCEDURE

A. *Three dimensional vector mean*

The attitude in space of a dipping plane can be conveniently portrayed by a vector of unit length which is erected perpendicular to the surface of such plane.

The average attitude of a given set of planes can be found through the following steps:

1) Consider all the planes to intersect at a given reference point.
2) Erect unit vectors perpendicular to each plane starting at the reference point.
3) Construct the vector sum of the unit vectors. (fig. 3, R_{xyz})
4) Derive the mean attitude of the set of planes, which is perpendicular to the attitude of the vector sum.

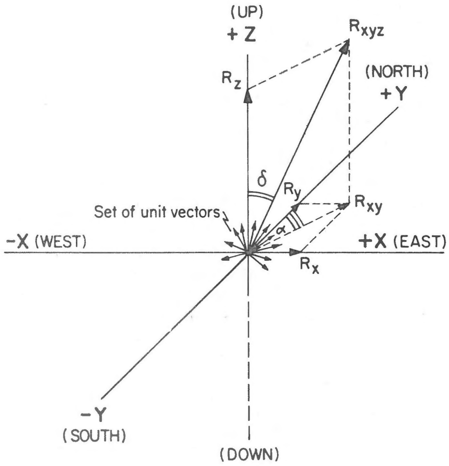
B. *Confidence limits of the vector mean, mean azimuth and mean dip angle*

Because dipmeter orientation data will always yield a vector sum and hence a mean azimuth and mean dip, even if meaningless random data are averaged, it is useful also to have a measure of uncertainty of the vector mean.

The range of uncertainty of a vector mean is dependent on the variance of the population from which the sample is drawn and on the size of the sample. A convenient way to describe such range of uncertainty is to give 95% confidence limits around the mean. Such confidence limits can only be estimated if a theoretical probability distribution adequately describing the data is available.

Although, in several cases, "goodness of fit" tests indicated significant "poor fit" for certain sets of dips, the "Fisherian distribution" (i.e. "spherical normal distribution", Fisher, 1953; Watson, 1966) appears to describe the data reasonably enough for the practical purpose of providing a yardstick of uncertainty of the vector mean. "Poor fit" to the theoretical distribution may commonly be explained by an admixture of extreme values due to various causes, e.g. random correlations of the dipmeter curves.

The distribution law assumes that data (if visual-



FORMULAS FOR CALCULATING THE 3-D VECTOR MEAN

(1) $R_x = \sum_{i=1}^{N} \sin \alpha_i \sin \delta_i$

(2) $R_y = \sum_{i=1}^{N} \cos \alpha_i \sin \delta_i$

(3) $R_z = \sum_{i=1}^{N} \cos \delta_i$

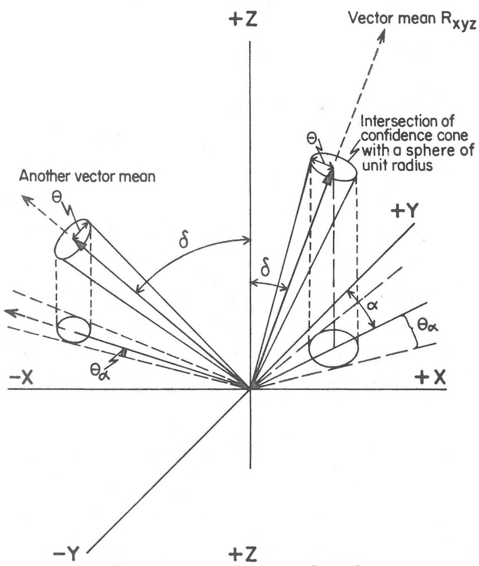
(4) $R_{xy} = \sqrt{R_x^2 + R_y^2}$

(5) $R_{xyz} = \sqrt{R_x^2 + R_y^2 + R_z^2}$ Vector sum

(6) $\alpha_{xyz} = \text{arc tan}(R_x / R_y)$ Mean azimuth

(7) $\delta_{xyz} = \text{arc tan}(R_{xy} / R_z)$ Mean dip angle

(8) N = Number of readings



FORMULAS BASED ON THE SPHERICAL NORMAL DISTRIBUTION

estimate of the precision constant K:

(9) $\hat{K} = \dfrac{N-1}{N-R_{xyz}}$

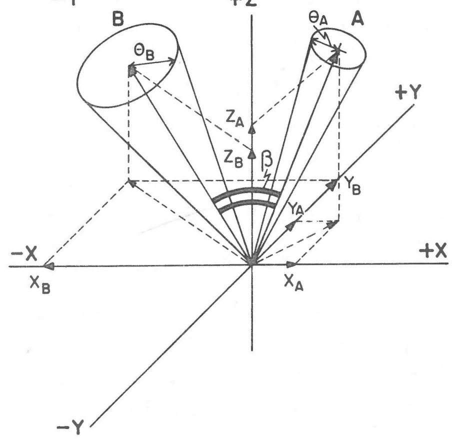
estimate of the semi-angle of confidence cone of the vector mean at probability level (1-P):

(10) $\hat{\theta} = \text{arc cos}\left[\dfrac{N}{R_{xyz}} - \dfrac{N-R_{xyz}}{R_{xyz}\, P^{\frac{1}{N-1}}}\right]$

estimate of the semi-angle of confidence of the azimuth:

(11) $\hat{\theta}_\alpha = \text{arc cos}\left[\dfrac{\cos^2\hat{\theta} - \cos^2\delta_{xyz}}{\sin \delta_{xyz}}\right]$ .... $\hat{\theta} \leq \delta_{xyz}$

(12) $\hat{\theta}_\alpha = 180°$ .... $\hat{\theta} > \delta_{xyz}$



ANGLE IN SPACE BETWEEN TWO VECTOR MEANS

(13) $\beta = \text{arc cos}\left[X_A X_B + Y_A Y_B + Z_A Z_B\right]$

where A and B are unit vectors coïnciding with the respective vector sums R_A and R_B of samples A and B

Rough approximation of the probability that the two samples A and B have been drawn from the same spherical normal distribution:

(14) $P(A=B) \sim \left[\dfrac{N_A + N_B - R_A - R_B}{N_A + N_B - R_{(A+B)}}\right]^{N_A+N_B-2}$

where $R_{(A+B)}$ is the vector sum of the combined sample.

Fig. 3
Summary of formulas.

ized as the end points of unit vectors) can cover the whole sphere, while the dipmeter readings are limited to a hemisphere only. The spread in directions is usually small, however, and no problems arise through this truncation.

In three dimensions, the confidence limit can be visualised as a cone around the mean vector and the 3-D confidence angle as the semi-angle of this cone. (fig. 3)

On a two-dimensional presentation (e.g. subsurface map) it is not possible to indicate this three-dimensional range of uncertainty graphically. Instead, it would be necessary to indicate the uncertainty in azimuth separately. It can be shown that the confidence limits of the azimuth are a function of a) the confidence limits of the 3-D vector mean and b) the mean dip angle (fig. 3). The larger the mean dip angle, the narrower are the confidence limits of the azimuth. However, at low mean dip angles the vertical Z axis commonly lies within the confidence cone. In such cases the confidence angles of the azimuth are 180°, and the range between the confidence limits covers the whole circle (0-360°).

In practice it appears that the azimuth confidence angles are rather pessimistic. This follows from the common observation that, in a vertical sequence of low dips, the mean azimuth of successive intervals can be remarkably constant although the azimuth-confidence angle may be 180° for all the intervals concerned (see fig. 4). It is important to realize that not all directions within the 95% confidence limits are equally likely, the mean azimuth being more probable than the azimuths indicated by the lower and higher confidence limits.

The confidence limits of the mean dip angle are calculated more easily because the 3-D confidence angle "theta" can be added to, or subtracted from the mean value. Negative values of the lower confidence limit (vertical Z-axis lies within the confidence cone) have to be interpreted as maximum dips in a direction opposite from the mean azimuth.

C. *Precision constant*

An estimate of the precision constant "K" is obtained almost as a by-product of the calculations on the confidence limits. It should be noted that the parameter K, unlike the confidence angle, is independent of the sample size, unless very small samples are involved. This parameter may be of considerable value to the sedimentologist, as the scatter of the dips appears to reflect the energy of the depositional environment, (G i l r e a t h et al, 1969). However, one should be careful in using this parameter as factors unrelated to environment may also influence K.

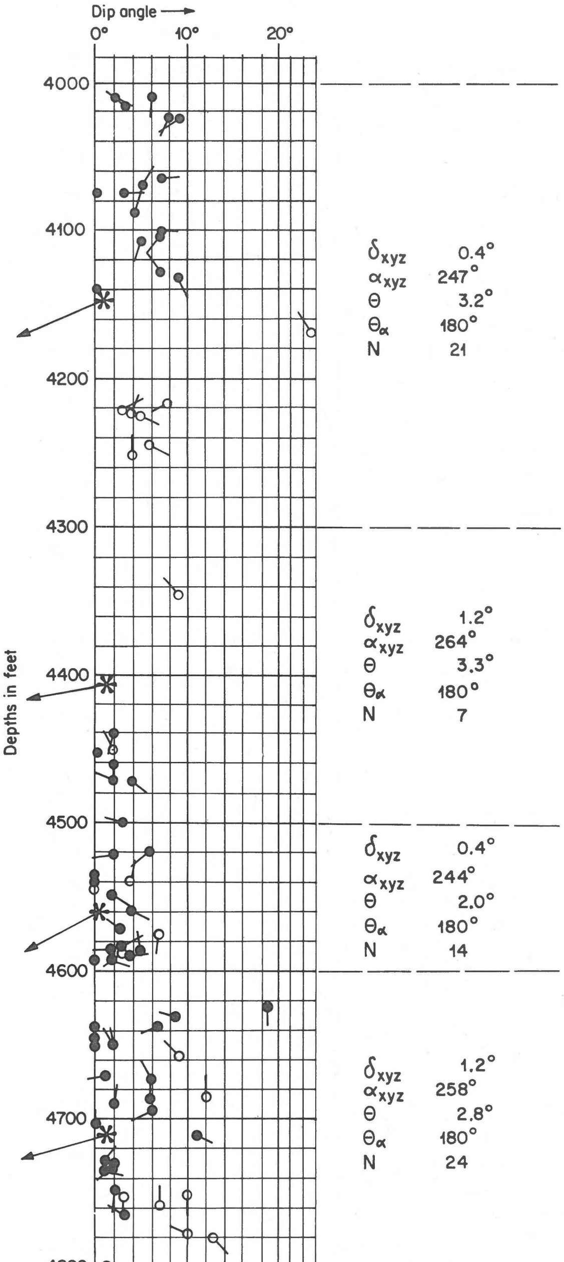


Fig. 4
Example of consistent mean azimuths at low dip angles.

SELECTION OF DATA

Assuming that the main objective of dipmeter interpretation is to determine structural dip, it is useful to eliminate irrelevant dips such as dips reflecting large scale cross bedding, grooving of the borehole wall, etc, which would require some form of interpretation preceding the statistical treatment. The sample of dips can be "cleaned" further by choosing only better quality readings, as indicated by the logging company, and/or by eliminating readings which are clearly spurious. The latter elimination can be achieved by automatically excluding dips deviating further than some prescribed angle from the calculated vector mean, and then recalculating this mean with the "cleaned" data only.

GROUPING OF DATA

The choice of depth intervals for which dips are to be averaged poses several practical problems. The easiest method would be to specify in advance that dips should be averaged every 50 feet or so.

In Nigeria, shales and finely interbedded sand/shale sequences usually yield better structural dip than sands (fig. 5). In carbonate sections good results are obtained in thin bedded limestones but poor results in dense massive limestones in which fractures may occur. Therefore a listing of the intervals in which structural dips are expected to predominate can be used instead of regular depth intervals. Moreover, the position of faults and unconformities are often known in advance from log correlation to avoid straddling such discontinuities with depth intervals. If available in time, the tadpole plot with the original data can provide additional information for an optimal choice of depth intervals.

In summary, some geological interpretation should precede the calculation of average dips if feasible.

Ideally the interval size should be related to the rate of change of the structural dip and azimuth with depth and the number of dipmeter readings necessary to produce a reliable average. The program is organ-

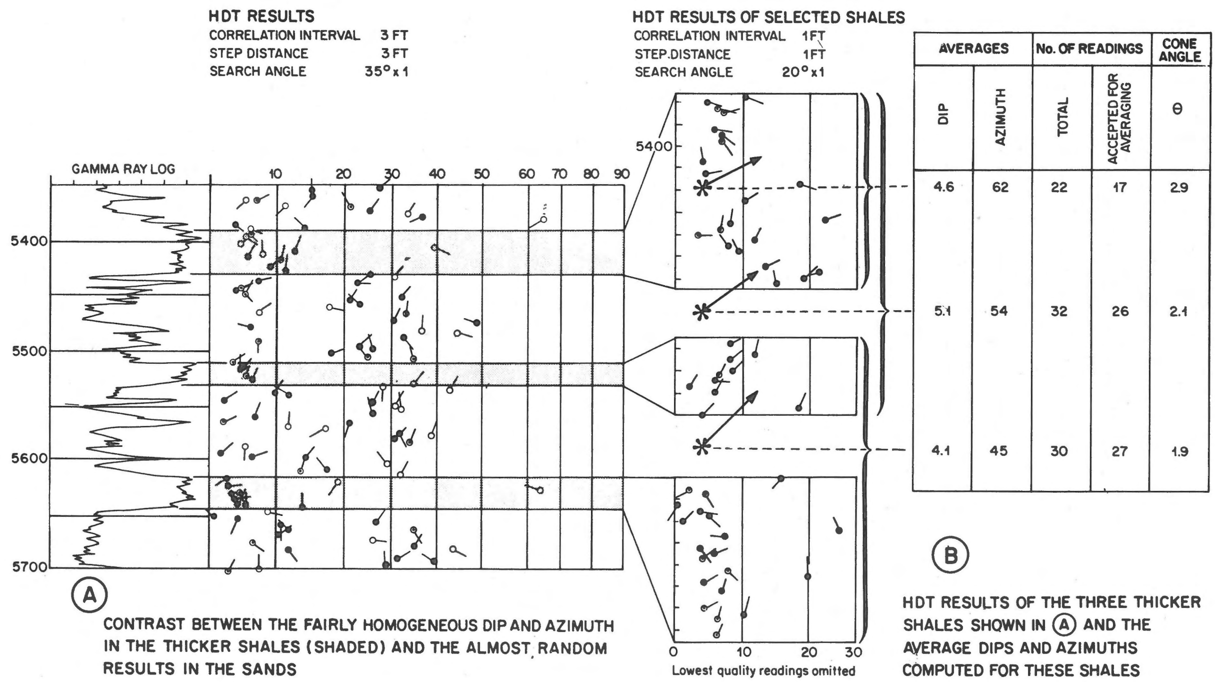


Fig. 5
Vector means of dips in a sand/shale sequence, Niger Delta. Shale dips evaluated only.

ized in such a way that if a certain interval contains too few dips to give a meaningful average it is combined with the underlying interval. This is only done, however, if a simple two-sample test indicates no significant difference in mean directions, (Watson & Williams, 1956, p. 351 formula 37). Calculation of the angle in space between successive vector means and testing the statistical significance of such differences results in a listing of depths where possible discontinuities (e.g. faults, unconformities) may occur.

DISPLAY OF RESULTS

The listing produced by the program gives depth intervals, midpoints, mean dip and azimuth, confidence limits and angular differences between successive means.

Because the 3-D averaging method has removed most of the unnecessary scatter of the 2-D tadpole plot it is quite convenient to plot the average dips at interval midpoints as before on the (2-D) tadpole plot prior to an interpretation. For this purpose it is convenient to use a condensed scale tadpole or azimuth and dip plot (say, 1:2500), instead of the original 1:200 or 1:1000 scales. An example from Nigeria is shown in Fig. 6.

The vector means could also be spotted on subsurface maps by a computer . The confidence limits of the azimuth would be a useful guide when contours are drawn.

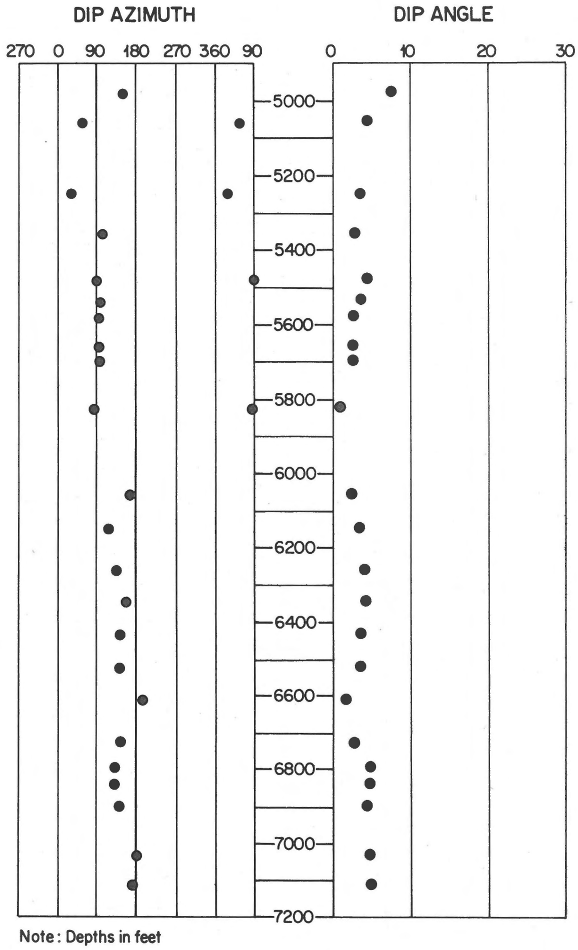


Fig. 6
Condensed azimuth/dip plot from a Niger Delta well based on shale dips only.

CONCLUSIONS

Provided some preliminary geological interpretation precedes the calculation procedure, the 3-D vector method considerably simplifies dipmeter interpretation.

Especially in low-dip areas, the dip angle values calculated by this method are commonly more realistic than estimates made on the basis of a tadpole plot (figs. 2, 4 and 5). Apart from the vector mean, the precision constant K is calculated which may be useful for the sedimentologist in environmental interpretation.

Finally, the difference between successive vector means is a measure of the rate of change of dip with depth and can reveal the presence of discontinuities, such as faults and unconformities which might otherwise remain undetected.

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REFERENCES

Fisher, R.A. (1953) – Dispersion on a sphere. Proc. Roy. soc. A, Vol. 217, p. 295-305.

Gilreath, J.A., J.S. Healy & J.N. Yelverton (1969) – Depositional environments defined by dipmeter interpretation. (Abstr.) Amer. Ass. Petrol. Geol. Bull. V. 53, No. 9, p. 2036.

Watson, G.S. (1966) – The statistics of orientation data. Journal of Geol. Vol. 74, No. 5, Part. 2, p. 786-797.

Watson, G.S. & E.J. Williams (1956) – On the construction of significance tests on the circle and the sphere. Biometrika, Vol. 43, p. 344-352.