GEOLOGIE EN MIJNBOUW VOLUME 48(1), p. 3-8 1969

THE PATHS OF GRAINS THROUGH WAVE-LIKE BED FORMS

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SUMMARY

From a knowledge of the topography of skin-friction lines, the paths of transported grains relative to the ground and to bed features are presented for different kinds of wave-like bed form encountered in aqueous flows. In the case of a sinusbed (antidunes), the height of the forms and their propagation velocity can be used without correction to measure the rate of downstream transport of sediment involved in the movement of the waves. The flow over a ripple or dune bed is separated, however, and within the forms there occurs a sediment stream moving backward relative to the external flow direction. The rate of downstream transport of sediment involved in movement of the waves is in these cases a net rate depending on the ratio of the thickness of the forward-moving and backward-moving sediment streams. Thus estimates of bedload transport rate based on the full heights of ripples or dunes can be in substantial error.

INTRODUCTION

Field and laboratory workers in the field of sediment transport experience much difficulty in making direct and accurate determinations of sediment transport rate in open channels. These difficulties are compounded when bedload is to be measured separately from suspension load. As regards bedload measurement, a number of indirect methods have therefore been devised. One of these, now being widely applied, rests on the idea that the bedload transport rate is measured by the rate at which wave-like bed forms are propagated beneath a flow, given the size and shape of the forms. Thus it is assumed that the bedload transport rate is equal to the rate of transport of sediment involved in the movement of the waves. Since this latter rate can be zero for a finite value of the true bedload transport rate, however, it is useful to distinguish the sediment involved in the movement of bed waves as the waveload. The chief wave-like bed forms requiring attention are ripples, dunes and what have been called antidunes (Kennedy, 1961; Simons et al., 1961, 1965a).

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The method depends on the differential equation of sediment transport for a two-dimensional wavy bed, as discussed by Bagnold (1954), Richardson et al. (1961), Simons et al. (1965b), Stein (1965), and Engelund and Hansen (1966, 1967). Referring to fig. 1, and without repeating the individual derivations, it can be deduced that

$$Q_b = \gamma V y + C, \tag{1}$$

in which Q_b is the local transport rate per unit of width of bedload material measured at a point of elevation y above a plane parallel to the x-direction, γ is the specific weight of the sediment in bulk, and V is the propagation velocity of the bed wave relative to the ground. The quantity C can be interpreted as that part of the bed material load not involved in the wave propagation. The waveload is obtained by setting $C = 0$, whence assuming that the sediment particles all move downstream and that the bed forms have triangular shapes, we have that

$$Q_W = \frac{\gamma V H}{2} \tag{2}$$

where Q_W is the mean waveload transport rate per unit width, H is the mean height of the bed waves, and V is the mean propagation velocity of the waves.

These expressions involve only the geometrical and kinematical properties of the bed waves, but it appears that their validity depends on an assumption, hitherto ignored or never made explicit, about the topography of streamlines and skin-friction

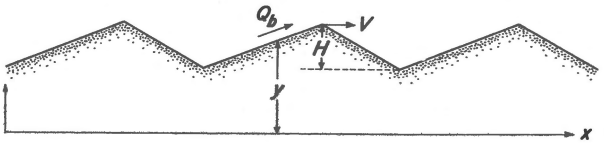


Fig. 1 - Definition diagram for two-dimensional sediment transport over a wave bed.

lines relevant to the flow over the bed. A streamline can be defined as a line which is tangent to the velocity vector of all fluid particles which lie on it. A skin-friction line is a "limiting streamline" and thus pertains to the shear stress vector at the flow boundary; it is a line along which bed material would travel. The assumption referred to is that the grains which make up the bed wave are propagated in the same direction as the general fluid flow to which the waves are adjusted. This assumption is invalid where there are regions of separated flow, such as persist in the flow over a ripple or dune bed.

SINUSBED

Gilbert (1914) gave the name antidune to sinusoidal bed waves more or less in phase with water waves that migrated in the opposite direction to flow. Later work showed that the waves could be propagated downstream, upstream or not at all, according to flow conditions. Simons et al. (1961) reserved the term antidune to those waves propagated opposite to flow, and to waves that were stationary gave the name standing waves. But Kennedy (1963) applied antidune to all bed waves in phase with surface waves, irrespective of the propagation direction. It is, however, open to question whether the bed and surface waves are always substantially in phase (Reynolds, 1965; Engelund and Hansen, 1966). Later, Engelund and Hansen (1966), pointing to a logical absurdity in the usage adopted by Kennedy, proposed to call the features a sinusbed. A positive sinusbed migrates in the direction of fluid flow, whereas a negative sinusbed travels opposite to flow. A stationary sinusbed is the standing wave of other authors.

The flow pattern relative to the ground over a sinusbed is shown in fig. 2. The streamlines in the body of the fluid, and the skin-friction lines which lie in the flow boundary, are all sinuous and unidirectional in the plane parallel to flow. Persistent regions of separation are absent, although separation does occur momentarily when the waves break (Kennedy, 1961, 1963; Simons et al., 1961; Middleton, 1965). Thus particles involved in the sediment transport ordinarily travel only downstream, along a skin-friction line such as AA'.

First consider the positive sinusbed AA' at time

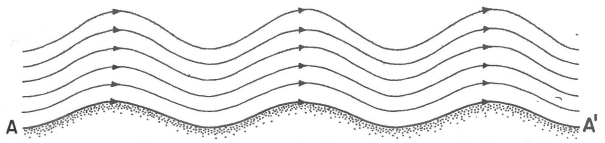


Fig. 2 - Two-dimensional fluid motion relative to the ground over a sinusbed.

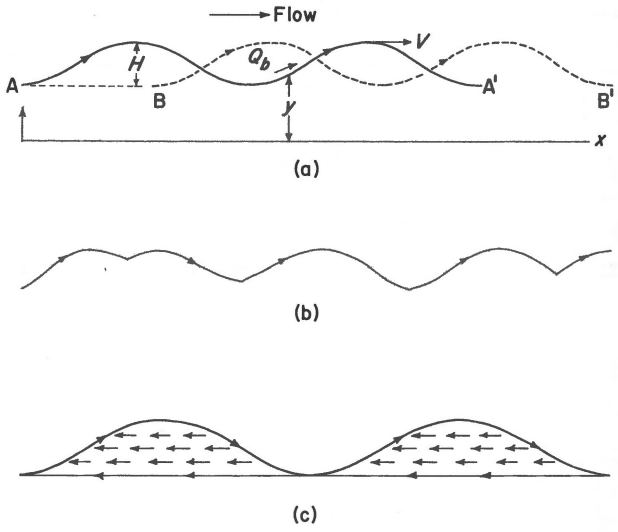


Fig. 3 - Two-dimensional sediment transport over a positive sinusbed:
(a) motion of bed form and definition diagram;
(b) schematic grain paths relative to ground;
(c) schematic grain paths relative to wave form.

T_1 (fig. 3a). Suppose that by some later time T_2 the waves have advanced to some downstream position BB'. This can only take place by the erosion of a quantity of sediment from the upstream face of each wave and the deposition of an equal quantity on the downstream face. The local transport rate Q_b is therefore a maximum at the wave crest, and since V is measured in the positive x-direction, we must measure y from a plane situated below the bed. The path relative to the ground of an individual particle partaking in the motion of the bed form would appear somewhat as sketched in fig. 3b. Relative to the waves, however, particles would

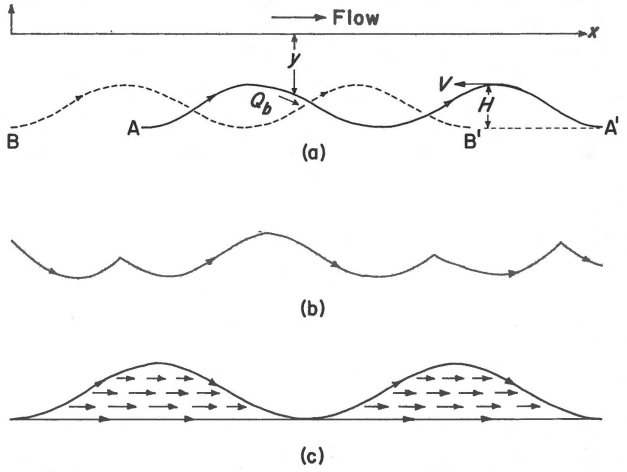


Fig. 4 - Two-dimensional sediment transport over a negative sinusbed:
(a) motion of bed form and definition diagram;
(b) schematic grain paths relative to ground;
(c) schematic grain paths relative to wave form.

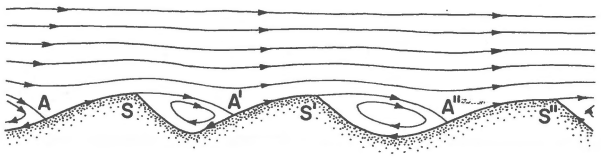


Fig. 5 - Two-dimensional fluid motion relative to ground over a ripple or dune bed.

follow paths represented by the arrows in fig. 3c.

The propagation velocity of a negative sinusbed (fig. 4a) is measured in the negative x-direction. Thus at time T_2 the waves occupy a position BB' which is further upstream than the initial position AA' at time T_1. The upstream movement of the waves requires that sediment is eroded from the downstream side of one wave and deposited on the upstream face of the next wave downstream. Clearly the local transport rate is now a maximum at the wave troughs but a minimum at the crests. But since the grains all travel in the downstream direction, we must measure y downward from a plane above the bed in order for eq. (1) to have correct signs. Relative to the ground, the paths of grains partaking in the movement of the waves appears as in fig. 4b. The grain paths shown in figs. 3b and 4b bear a mirror-image relationship to each other. Relative to the waves, however, the particles moving in a negative sinusbed travel downstream along paths through the forms (fig. 4c). The corresponding field of grain flow for the positive sinusbed showed an upstream movement of grains.

RIPPLES AND DUNES

Ripple and dune beds are common products of unidirectional flow in open channels. The pattern of fluid motion relative to the ground associated with ripples and dunes is known in general terms (fig. 5) from the work of Darwin (1884), Reineck (1961), Simons et al. (1961, 1965a), Raudkivi (1963, 1966) and Jopling (1964, 1967). In the lee of each bed wave is a recirculating separation bubble (loosely called an eddy or roller) divided from the external stream by a separation streamline, e.g. SA' or S'A". The streamlines within each bubble are closed loops, whereas those for the external flow are sinuous and unidirectional. Thus the fluid in the lower part of the bubble moves in the opposite direction to the external flow. We consequently find that skin-friction lines, traced along the bed, display points of convergence alternately with points of divergence. The points of divergence are points of attachment for streamlines from the body of the fluid which rejoin the bed. Observation teaches that the attachment does not occur at the lowest point of the ripple or dune trough, where the bed slope is zero, but at some point on the tilted, upstream surface of each bed feature (e.g. Reineck, 1961; Jopling, 1967). Strong erosion occurs at such points, the sediment being moved away upstream as well as downstream.

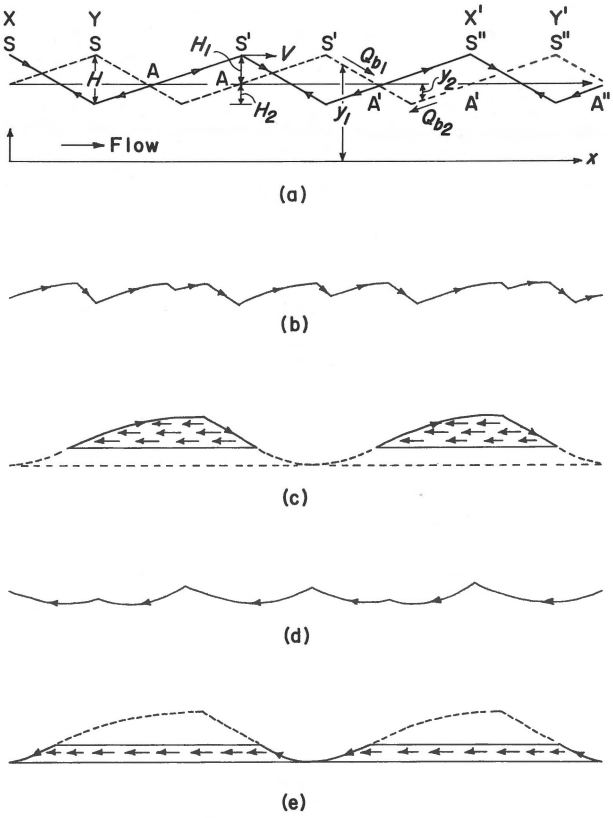


Fig. 6 - Two-dimensional sediment transport over a ripple or dune bed:
(a) motion of bed form and definition diagram;
(b) schematic grain paths relative to ground for upper sediment stream;
(c) schematic grain paths relative to wave for upper sediment stream;
(d) schematic grain paths relative to ground for lower sediment stream;
(e) schematic grain paths relative to wave for lower sediment stream.

Thus, of the sediment taking part in the propagation of the forms, a quantity moves over the bed in the opposite direction to the external fluid stream, whilst another quantity moves with the external flow. Observation also teaches that the sediment moving against the external stream contributes substantially to a bottomset deposit which accumulates below the slip face deposits of the bed feature.

Consider the ripple or dune bed shown in fig. 6a. At an initial time T_1 the bed occupies a position XX', but at some later time T_2 is located at YY'. Movement has occurred because of erosion from the upstream face of each feature and deposition on the downstream face. Now a line drawn parallel to the x-direction through the attachment points A,

A' and A" divides the train of bed waves into two regions distinct as regards the paths of grains moving over and through the forms. In the portion of each form above the line through the attachment points, the grain paths relative to the ground are as in fig. 6b and relative to the form as in fig. 6c. But for that portion of each form below the line through the attachment points, we have grain paths relative to the ground as sketched in fig. 6d and those relative to the form shown in fig. 6e.

It is easy to see from the definitions given in fig. 6a and the previous discussion that for a ripple or dune bed we must write for the stream of sediment above the attachment points

$$Q_{W_1} = \frac{\gamma V H_1}{2} \tag{3a}$$

and for the stream below

$$Q_{W_2} = -\frac{\gamma V H_2}{2} \tag{3b}$$

Consequently, the net downstream waveload transport rate given by the propagation velocity and scale of the bed forms is

$$Q_W = Q_{W_1} + Q_{W_2} \tag{4}$$

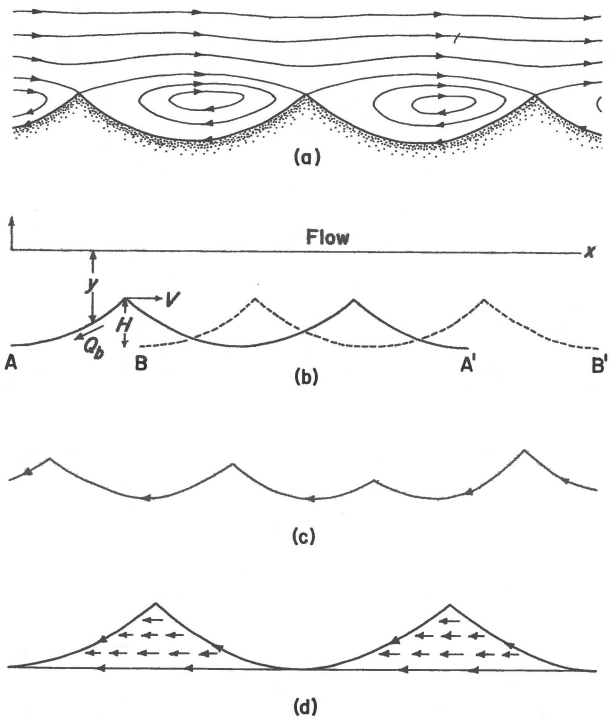


Fig. 7 - Two-dimensional sediment transport over a hypothetical wavy bed on which separation and attachment points coincide:
(a) possible fluid motion;
(b) motion of bed form and definition diagram;
(c) schematic grain paths relative to ground;
(d) schematic grain paths relative to wave form.

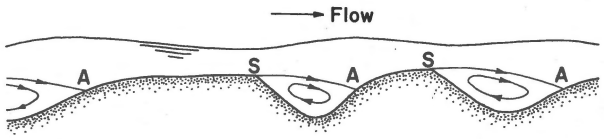


Fig. 8 - Two-dimensional fluid motion relative to the ground for a ripple or dune bed at large relative roughness.

in which Q_{W_2} is from eq. (3b) a negative quantity.

This result helps to explain the considerable scatter found in data on transport rates based on ripple or dune height, as well as discrepancies observed between calculated and measured rates (Simons et al., 1965b; Stein, 1965). Experiments made by the author have shown that H_2 appearing in eq. (3b) is not uncommonly 20-30 percent of the full height of the bed form.

Finally, as a purely hypothetical but instructive case, consider the situation when attachment and separation coincide on a wavy bed (fig. 7a). If as sketched in fig. 7b a representative portion of the bed lies at AA' at time T_1, but at BB' at some later time T_2, then the waves propagate downstream by erosion of sediment from each upstream face and deposition on the downstream slope lying immediately upstream. The local transport rate is thus a maximum in the troughs of the waves, as was found for a negative sinusbed. Grain paths relative to the ground and to the waves appear as in figs. 7c, d. A condition not too far removed from that of fig. 7 is often encountered where ripples or dunes exist in a flow of large relative roughness, about 1 or larger. Attachment is found to occur high up on the backs of the flat-topped bed forms (fig. 8).

CONCLUSION

A method of measuring bedload transport rate that has been widely used depends on knowledge of the propagation velocity and scale of wavelike bed forms. The method has been applied using sinusbeds as well as ripple and dune beds. But in the case of ripple or dune beds, account has to be taken of a stream of sediment which moves upstream in response to the recirculating separated flow captive in the lee of each bed feature. This effect, hitherto overlooked, has been discovered during the course of an investigation into the paths of grains as they move over and through bed forms. That a wave form migrates with a stream is now seen to be no guarantee that the sediment particles are also moving downstream.

It would be absurd, of course, to infer from the analysis that there exist transport systems in which sediment is permanently moving upstream. However, it is very probable that in natural systems some upstream movement does occur for short periods and for limited distances. And since trans-

port rates are generally measured at fixed stations and over short time intervals, attempts should be made to take the effect into account. The analysis is not invalidated when bed forms are strongly three-dimensional, though for simplicity it was developed for two-dimensional conditions.

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